

Melting Away

A back-of-the-envelope look at
sea level rise due to global warming.



Shoshana Sommer
New York University
shoshana.sommer@gmail.com
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(Image from *Global Warming and Climate Change*¹)

Author's Note

Sea level change is like a very large puzzle. The model with which climatologists work has a great number of tiny puzzle pieces which are challenging to fit together properly. The work here is instead built on far fewer, larger puzzle pieces (like the ones for ages 6 and under rather than for ages 7 through adults). Yet when the pieces are fitted together, it shows us practically the same picture. And the act of putting it together allows us to gain a greater understanding of and subsequent appreciation for the final product.

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Global climate change is an evident and growing threat, forcing itself into the public awareness. Naturalists worry about large scale extinctions, meteorologists point to out-of-season storms, agronomists consider the deleterious effects on crops, social scientists fret over the fate of poorer nations, and the average person scratches his head and wonders what it all means.

Journalists who digest the words of scientists are unable to adequately communicate to the public the magnitude of the situation. The problem is that the significant understandings to be communicated do not depend on the specifics of any one climate model, nor on any particular set of observations.² Yet precision and significance are not the same, and the overriding message of climate models is that the problem is real regardless of the details. And getting stuck in scientific details is a dead end for the majority of the population.

Yet without too many details, it is possible for the average person to gain a fundamental understanding of what is happening. And a person who has taken only a few courses in physics or math can actually arrive at a reasonable estimate of how much the sea level will rise if the average global temperature increases by three degrees over the next century. So while the more precise estimates of climatologists will likely remain a mystery to the majority of the population, there are a few back-of-the-envelope calculations that allow for gaining insight into the results produced by complex climate models.

Global Warming

The scientific community has come to the consensus opinion that an increase in atmospheric carbon dioxide has resulted in a lower percentage of reradiated radiant heat escaping into space. The result is that the world is warming up, tending toward higher equilibrium temperatures, which will continue to increase if the carbon dioxide level does not stabilize. The raised temperature is responsible for two major kinds of contributions to changes

in ocean volume. The first is the expansion of ocean waters as a result of temperature increases, and the second is the direct net addition or subtraction of quantities of water and ice to the overall volume of the ocean.³

The Significance of CO₂

Although global warming is actually a very complex process, it can be boiled down to a few basics. Rising global temperatures are the result of increased carbon dioxide in the atmosphere. Initially, solar radiation from the sun penetrates Earth's atmosphere to varying degrees. This initial solar radiation has relatively shorter wavelengths, and therefore carries greater energy on average, than the other forms of radiant energy transfer with which we are concerned.

At the planet's surface, there is another chance for light to be reflected or absorbed. Some of the incoming solar, or shortwave, radiation is actually absorbed and then reradiated as terrestrial, or longwave, radiation. Much of this outgoing radiation is in the form of thermal radiation, or heat. Other energy is globally redistributed through evaporative and other processes. Thermal radiation, with its longer wavelength in the infrared spectrum, has far more difficulty penetrating the atmosphere than visible radiation with its shorter wavelength. In particular, carbon dioxide in the atmosphere actually absorbs these infrared wavelengths, acquiring much of the energy and increasing the average atmospheric temperature.^{4,5}

Maintaining a specific global temperature requires that energy gained from solar radiation be equal to energy lost from the atmosphere back into space.⁶ With the increasing presence of carbon dioxide molecules in the atmosphere, a smaller percentage of the total energy is radiated outward and back into space, and therefore the average global temperature rises. This balance can be tipped even with small increases in the total amount of atmospheric carbon dioxide. Scientists point to automobiles, home heaters, electrical plants, and other fossil fuel burning processes as the biggest carbon dioxide producers.

Thermal Expansion

In a more expansive physical definition, thermal expansion refers to the tendency of matter to increase in volume when heated (the decrease in volume as ice undergoes the phase change to water is addressed later).^{7,8} This change is directly proportional to the temperature change, and can be quantified by a constant particular to a material and known as the coefficient of thermal expansion. Unlike fresh water, whose density is greatest several degrees above its freezing temperature, salt water is at its most dense at its freezing temperature, which is actually just below zero degrees Celsius.⁹ This suggests that all of the ocean is susceptible to a temperature induced expansion. As the average global temperature increases, ocean waters become less dense, and the volume they occupy increases, resulting in a net sea level rise.^{3,10}

While the equation for the thermal expansion is a simple linear proportionality, its application to the ocean involves complexities. Some result from empirical data regarding the effective depth for thermal mixing at the surface of the ocean. Others involve the temperature variations that exist at different depths and locations, and must be smoothed over for the sake of an order of magnitude calculation. We can also simplify the calculations by making the order of magnitude appropriate approximation the changes in ocean depth will not be greatly affected by changes in surface area.

Question: Can we reasonably make the order of magnitude approximation that changes in ocean depth will not be greatly affected by the assumption that the ocean surface does not change as sea levels rise?

1. A glance at the model and relevant formulas

Given:

SA_1 = Surface Area 1

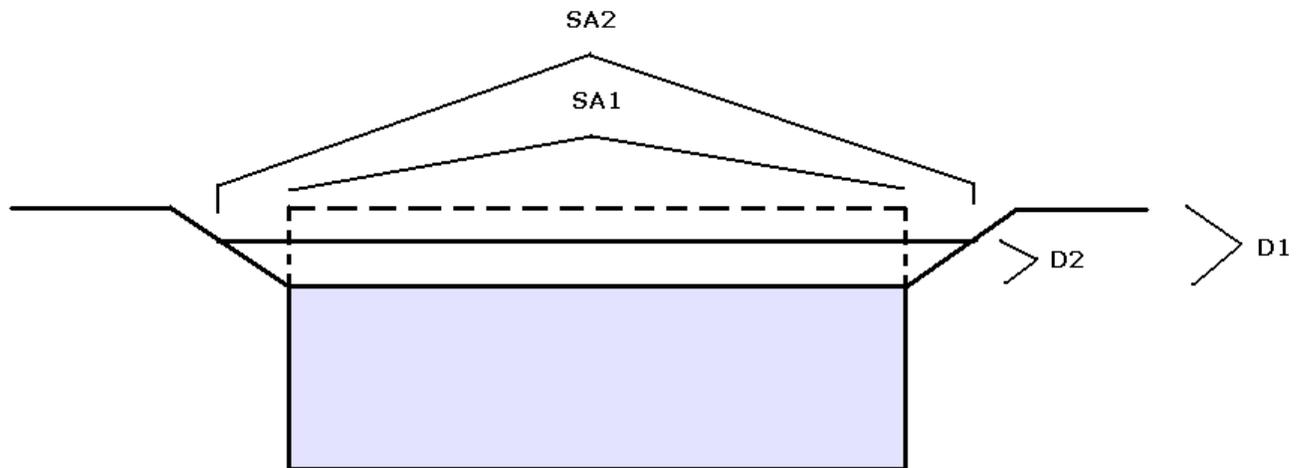
SA_2 = Surface Area 2

D_1 = Depth 1

D_2 = Depth 2

V = Volume

Cross-section of a simplified ocean:



$$[(SA_1)D_2] + [\frac{1}{2}(SA_2 - SA_1)][D_2] = D_2[\frac{1}{2}(SA_1) + \frac{1}{2}(SA_2)] = \frac{1}{2}D_2[SA_1 + SA_2]$$

$$\frac{1}{2}D_2(SA_1 + SA_2) = V = (SA_1)(D_1)$$

$$D_2/D_1 = (2SA_1)/(SA_1 + SA_2)$$

Or

$$D_1/D_2 = (SA_1 + SA_2)/(2SA_1)$$

2. Defining delta such that $SA_2 = SA_1(1 + \Delta)$, and substituting into the expression $D_1/D_2 = (SA_1 + SA_2)/(2SA_1)$

$$D_1/D_2 = (SA_1 + [SA_1(1 + \Delta)]) / (2SA_1) = [2SA_1 + (SA_1)(\Delta)] / (2SA_1) = 1 + \Delta/2$$

Answer: For example, assuming that the ocean's surface area increases by 5 % as the sea level rises (in other words, delta = 0.05), then the error from using D1 instead of D2 would be only 2.5 %. Thus relative errors in ocean surface area produce even smaller relative errors in depth.

END NOTES: Simple observation shows us that when the tide rises, the surface area of the flooded coastlines is not appreciable as compared to the surface area of the Earth's oceans (intertidal zone).

Question: How much will the sea level rise as a result of expanding ocean waters if the average temperature of the planet increases by 3 degrees over 100 years?

1. Calculating the depth of the top 10 % of the ocean, which is most affected by temperature change

Given⁷:

Upper 10% of the ocean is most affected by initial mixing processes.

Mean ocean depth (D_i) = 3794 m

Mean ocean depth x 0.10 = Depth of the top 10 % of the ocean

Calculations:

$$(3794 \text{ m})(0.10) = 379.4 \text{ m}$$

2. Calculating sea level rise as a result of thermal expansion

Given⁸:

$$\Delta \text{Volume} = \beta V_i \Delta T$$

$$\Delta \text{Volume} = (SA)(\Delta D)$$

Where

ΔV = change in volume

β = coefficient of expansion (a constant) = $1.5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ based on the average temperature of the upper waters at $\sim 10^\circ\text{C}$ and an average salinity of 35 ppt (10^{-3})^{7,11}

V_i = initial volume

ΔT = change in temperature = 3°C

SA = surface area

ΔD = change in depth (unknown)

For the purpose of simplification, assume

the initial surface area (SA_i) = final surface area (SA)

(sufficient for an order of magnitude calculation, although SA_i is actually $< SA_f$).

$$(SA)(\Delta D) = (\beta)(D_i)(SA)(\Delta T)$$

$$(\Delta D) = (\beta)(D_i)(\Delta T)$$

Where

D_i = initial depth = 379.4 m

Calculations:

$$\Delta D = (1.5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1})(379.4 \text{ m})(3^\circ\text{C}) = 0.17 \text{ m} = 17 \text{ cm}$$

Answer: The sea level will rise by 17 cm as a result of thermal expansion over 100 years and with a 3 degree temperature increase.

END NOTES: The Intergovernmental Panel on Climate Change (IPCC), 2001, has calculated that sea level will rise about 16 cm in 100 years as a result of the increasing temperature.¹² In fact, sea level may actually increase in some areas (the poles) and decrease in others. In addition to temperature, time is also a relevant factor in the real world, because, while it may take either 5 years or 100 years for a specific change in atmospheric temperature to occur, additional time is required for the ocean water to come to any kind of temperature equilibrium with the atmosphere. In this calculation, time was accounted for by limiting the thermal expansion to the top 10 % of the ocean (thermal delay).⁷

Mass Balance

There are actually three major factors involved in determining the change in sea level as a result of the direct net addition or subtraction of quantities of water and ice to the overall volume of the ocean. The first is the increased melting of mountain glaciers and small ice caps around the globe. The second, which is possibly the most publicized, is the melting of Greenland. And a third significant factor in calculating sea level changes over the next one hundred years involves Antarctica, whose initial reaction to temperature increase will act to counter the first two processes.¹³

These calculations begin with a relatively simple yet very important empirical value known as the specific mass balance.¹⁴ In a general model, mass balance equals what goes into a system minus what goes out of the system, thus giving a net value for what has been gained or lost by the system. In the world of climate studies, this value refers to the net annual gain or loss of ice and snow mass at the surface of a glacier. More specifically, a glacial mass balance equals the difference, over time, between accumulation and ablation (precipitation and melting). Because accumulation predominates in the winter, and ablation in the summer, the natural smallest unit of time over which to measure and average these opposing processes is the year.¹⁵

A glacial sheet is naturally treated as a single geographical system, but in the short term its area can be considered as a scale factor to be removed, with regional variations absorbed into an average value true for the entire glacier. When known as the specific mass balance, this value is determined per unit area of the glacier's surface, and is averaged over the entire glacier. Taking away the reliance on area, the specific mass balance is actually the average depth of ice lost or gained per unit time (ie. per year).

A glacier with a sustained positive mass balance is growing in size, while a sustained negative mass balance implies the glacier is retreating, or losing mass. This makes mass balance values, which are empirically determined, significant for evaluating the dynamic state of global ice. Mass balance determinations made using satellites also include the effects of ice slippage from the land into the sea, and not just the effects of melting.

Another value of importance is the mass balance sensitivity, which is the change in specific mass balance per Celsius degree of temperature change. With this value, the snow and ice depth being lost or gained per unit time *and* its change per degree of temperature increase is known. In other words, the effects of global warming on ice depth, the result of average temperature increases over time, can be estimated in terms of specific mass balance and mass balance sensitivity values.¹⁵

Finally, using first an estimate for the density change involved in the transformation of ice into water, and then by the use of a conversion factor given by the ratio of the areas for the glacial sheet and the ocean, we can generate a separate equivalent sea level contribution for any body of ice. As with thermal expansion calculations, we can simplify the calculations immensely by making the back-of-the-envelope appropriate approximation that the ocean surface area does not change as water volume increases.

The combination of the mass balance sensitivity and the rate of temperature change with time gives us a rate per unit time value reflecting changes in the specific mass balance. In the short term, barring unforeseen nonlinear processes, we can assume that the temperature change over the next century will increase linearly with time, which seems to be consistent with the results of the far more detailed climate simulations being used. In addition, making the assumption that the mass balance sensitivity remains approximately constant implies a mass balance that changes linearly with time. This greatly simplifies calculating the total contribution to the sea level rise. It is important to understand, however, that in reality unforeseen processes could result in highly nonlinear changes. Thus order of magnitude estimates of sea level rise could merely be a minimum.

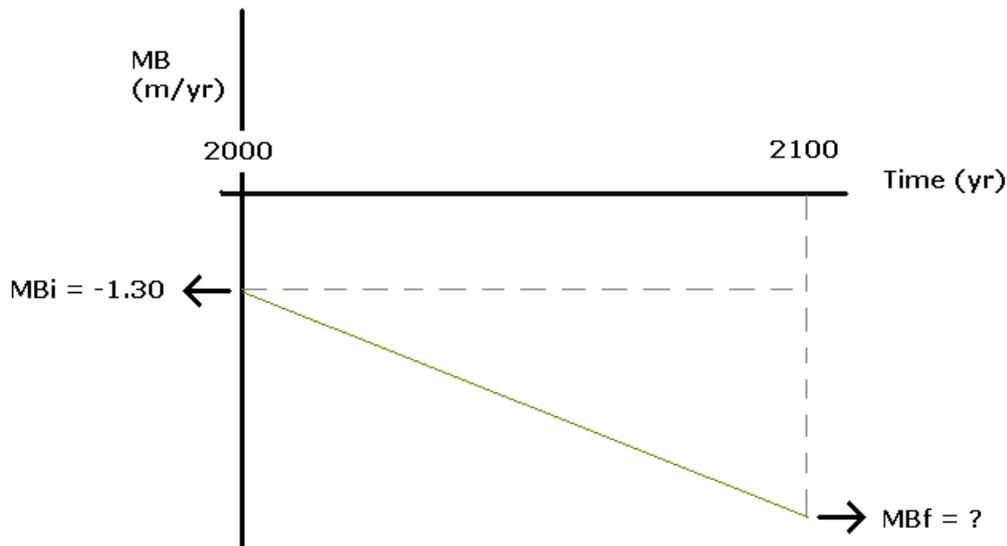
More detailed models actually imply larger local changes in temperature in the polar regions as compared to the overall global change in average temperature. Simulations and other evidence indicate that temperature increases may actually be more than twice as large in the polar regions as the three degrees being considered here.¹⁶ Therefore a three degree increase in temperature at the polar regions actually represents a minimum.

A. MOUNTAIN GLACIERS AND ICE CAPS

Question: What will mountain glaciers and ice caps contribute to the rise in sea level if the average temperature of the planet increases by 3 degrees over 100 years?

Initial Mass Balance (MB_i) = (accumulation + ablation)/year [m/yr] \approx -0.130 m/yr

Mass Balance Sensitivity¹⁷ (MB_s) = MB/°C \approx -0.35 m/yr/°C



We will assume that temperature, and thus MB sensitivity, will change linearly with time.

1. Calculating the final mass balance

Given:

$$MB_f = MB_i + (MB_s)(\Delta T)$$

Calculations:

$$MB_f = (-0.130 \text{ m/yr}) + (-0.35 \text{ m/yr/}^\circ\text{C})(3^\circ\text{C}) = -1.135 \text{ m/yr}$$

2. Calculating the depth of ice lost over 100 years as a result of MB change (Area 1)

Given:

The area between the graph and the axis = total depth of ice lost due to melting.

$$\text{Area 2} = \Delta D_2 = \frac{1}{2}[(MB_f - MB_i) \times 100 \text{ years}]$$

Calculations:

$$\Delta D_2 = \frac{1}{2}[(-1.135 \text{ m/yr}) - (-0.130 \text{ m/yr})]100 \text{ yrs} = -50.25 \text{ m}$$

3. Calculating the SLR over 100 years at a constant MB of -0.130 m/yr (Area 2)

Given¹⁷:

With a mass balance (MB_i) of - 0.130 m/yr, sea level rise (SLR) per year is approximately -0.19 mm/yr

The simple model described below, in which the area of the ice is assumed constant, would give a similar value.

$$M_{ice}/yr = (MBi)(SA_{ice})(Density_{ice})$$

$$M_{water}/yr = (SLR/yr)(SA_{ocean})(Density_{water})$$

$$M_{ice}/yr = M_{water}/yr = (MBi)(SA_{ice})(Density_{ice}) = (SLR/yr)(SA_{ocean})(Density_{water})$$

$$(SLR/yr) = (MBi)[(SA_{ice})/(SA_{ocean})][(Density_{ice})(Density_{water})]$$

Where

M_{ice} = Mass of ice lost by ice caps and glaciers
 M_{water} = Mass of water gained by the ocean
 SA_{ice} = Ocean surface area
Density_{ice} = Density of ice

Calculations (using the above SLR/yr of -0.19 mm/yr):
(-0.19 mm/yr)(100 yr) = 19 mm SLR

4. Calculating the total SLR as a result of MB change (continuing from section 2)

Given¹⁷:

MBi causes a sea level rise (SLR) each year of 0.19 mm/yr

Calculations:

$$(\Delta D/SLR) = (13 \text{ m}/19 \text{ mm}) = (50.25 \text{ m}/\Delta D)$$

$$\Delta D = 73 \text{ mm}$$

$$+ 19 \text{ mm (from #3)} = 92 \text{ mm} = 9.2 \text{ cm}$$

Answer: Sea level will rise by 9.2 cm as a result of the melting of mountain glaciers and ice caps over 100 years and with a 3 degree increase in temperature.

END NOTES: Raper and Brathwaite calculated a maximum sea level rise of 10.6 cm, while the IPCC, 2001, calculated a sea level rise of about 7.7 cm.^{17,12}

B. GREENLAND

Question: What will the Greenland ice sheet contribute to the rise in sea level if the average temperature of the planet increases by 3 degrees over 100 years?

***NOTE: I convert all rates/values to their equivalent sea level rise (SLR) values because the IPCC mass balance values for Greenland are in terms of the corresponding contribution to sea level increase rather than Greenland ice sheet decrease.¹² Rather than convert these values back to their equivalents in terms of ice melt, and then later convert the final values to sea level rise, I decided to work directly in terms of that component of sea level rise attributed to Greenland melting.**

1) Calculating the ratio between the densities of water and ice

Given:

$$Density_{water} / Density_{ice} = \text{Density ratio}$$

Where⁸

$$Density_{water} = 999 \text{ kg/m}^3$$

$$Density_{ice} = 917 \text{ kg/m}^3$$

Calculation:

$$999 \text{ kg/m}^3 / 917 \text{ kg/m}^3 = 1.089 = \text{Ratio of the density of water to the density of ice}$$

2) Calculating the rate of water volume increase from an empirical value for the rate of ice volume decrease**Given:**

$$(\text{Rate of ice loss}) / (\text{Ratio of ice to water}) = \text{Rate of water volume increase}$$

Where¹⁸

$$\text{Rate of ice loss} = -150 \text{ km}^3/\text{yr}$$

Calculation:

$$150 \text{ km}^3/\text{yr} / 1.089 = 137.741 \text{ km}^3/\text{yr} = \text{Rate of water volume increase}$$

3) Calculating the mass balance of Greenland in terms of sea level rise**Given:**

SLR = Sea Level Rise

$$10^{-7} \text{ km} = 0.1 \text{ mm}$$

Rate of water volume increase / SA_{ocean} = Mass Balance of Greenland in terms of SLR (= rate of SLR)

Where¹⁹

$$SA_{\text{ocean}} = 361 \times 10^6 \text{ km}^2$$

Calculation:

$$(137.741 \text{ km}^3/\text{yr}) / (361 \times 10^6 \text{ km}^2) = 3.81554 \times 10^{-7} \text{ km}/\text{yr}$$

$$\text{Mass Balance of Greenland (in terms of SLR)} = 0.38 \text{ mm}/\text{yr}$$

4) Calculating final mass balance in terms of SLR (= rate of SLR)**Given:**

$$MB_i + (MB_s)(\Delta T) = MB_f$$

Where

ΔT = Change in Temperature = 3°C

MB_i = Initial Mass Balance = $0.38 \text{ mm}/\text{yr}$

(calculated above)

MB_s = Mass Balance Sensitivity = $0.9 \text{ mm}/\text{yr}/^\circ\text{C}$ ¹²

MB_f = Final Mass Balance (after 100 years and a 3 degree temperature increase)

All of the above values are in terms of SLR, as this was how the empirical data concerning Greenland's mass balance values was presented by the IPCC.

Calculation:

$$0.38 \text{ mm}/\text{yr} + (0.9 \text{ mm}/\text{yr}/^\circ\text{C})(3^\circ\text{C}) = 0.65 \text{ mm}/\text{yr} = \text{Final mass balance in terms of SLR}$$

5) Calculating the change in depth of the ocean (SLR)**Given:**

$$\Delta D = \frac{1}{2}[(MB_f - M_{bi}) \times (\text{Change in Time})] + [(\text{Change in Time}) \times M_{bi}] = \text{Sea Level Rise}$$

where

ΔD = Change in depth of the ocean due to the mass balance change over 100 years and with a 3 degree local temperature increase (sea level rise)

All other values have already been calculated or given.

Calculation:

$$\Delta D = \frac{1}{2}[(0.65 \text{ mm/yr} - 0.38 \text{ mm/yr}) \times 100 \text{ yrs}] = 13.5 \text{ mm}$$

$$+ (100 \text{ yrs} \times 0.38 \text{ mm/yr}) = 51.5 \text{ mm} = 5.15 \text{ cm}$$

Answer: Sea level will rise by 5.15 cm as a result of Greenland's ice sheet melting over 100 years and with a 3 degree increase in temperature over 100 years.

END NOTES: According to the Max Planck Institute for Meteorology, Greenland will contribute an average of 4 cm to the total sea level rise over the next 100 years.²⁰ Other estimates, as well as the estimated Max Planck Institute range of sea level rise due to Greenland's melting ice, include my calculated answer.

C. ANTARCTICA

Question: What will the Antarctic ice sheet contribute to the rise in sea level if the average temperature of the planet increases by 3 degrees over 100 years?

***NOTE: I convert all rates/values to their equivalent sea level rise (SLR) values because the IPCC mass balance values for the Antarctic are in terms of the corresponding contribution to sea level increase rather than Antarctic ice sheet decrease. Rather than convert these values back to their equivalents in terms of ice melt, and then later convert the final values to sea level rise, I decided to work directly in terms of that component of sea level rise attributed to the Antarctic melting.**

1) Calculating the ratio between the densities of water and ice

Given:

$$\text{Density}_{\text{water}} / \text{Density}_{\text{ice}} = \text{Density ratio}$$

Where⁸

$$\text{Density}_{\text{water}} = 999 \text{ kg/m}^3$$

$$\text{Density}_{\text{ice}} = 917 \text{ kg/m}^3$$

Calculations:

$$999 \text{ kg/m}^3 / 917 \text{ kg/m}^3 = 1.089 = \text{Ratio of the density of water to the density of ice}$$

2) Calculating the rate of water volume increase from an empirical value for the rate of ice volume decrease

Given:

$$\text{Rate of ice loss} / \text{Ratio of ice to water} = \text{Rate of water volume increase}$$

Where¹⁸

$$\text{Rate of ice loss} = - 150 \text{ km}^3/\text{yr}$$

Calculations:

$$150 \text{ km}^3/\text{yr ice} / 1.089 = 137.741 \text{ km}^3/\text{yr} = \text{Rate of water volume increase}$$

3) Calculating the mass balance of the Antarctic in terms of sea level rise**Given:**

SLR = Sea Level Rise

$$10^{-7} \text{ km} = 0.1 \text{ mm}$$

Rate of water volume increase / SA_{ocean} = Mass Balance of the Antarctic in terms of SLR (= rate of SLR)

Where¹⁹

$$SA_{\text{ocean}} = 361 \times 10^6 \text{ km}^2$$

Calculations:

$$(137.741 \text{ km}^3/\text{yr}) / (361 \times 10^6 \text{ km}^2) = 3.81554 \times 10^{-7} \text{ km}/\text{yr}$$

$$\text{Mass Balance of the Antarctic (in terms of SLR)} = 0.38 \text{ mm}/\text{yr}$$

4) Calculating final mass balance in terms of SLR (= rate of SLR)**Given:**

$$MB_i + (MB_s)(\Delta T) = MB_f$$

Where

ΔT = Change in Temperature = 3°C

MB_i = Initial Mass Balance = $0.38 \text{ mm}/\text{yr}$
(calculated above)

MB_s = Mass Balance Sensitivity = $-0.48 \text{ mm}/\text{yr}/^\circ\text{C}$ ¹³

MB_f = Final Mass Balance (after 100 years and a 3 degree temperature increase)

All of the above values are in terms of SLR, as this was how the empirical data concerning the Antarctic's mass balance values was presented by the IPCC.

Calculations:

$$0.38 \text{ mm}/\text{yr} + (-0.48 \text{ mm}/\text{yr} / ^\circ\text{C})(3^\circ\text{C}) = -1.06 \text{ mm}/\text{yr}$$

= Final mass balance in terms of SLR

5) Calculating the change in depth of the ocean (SLR)**Given:**

$$\Delta D = \frac{1}{2}[(MB_f - MB_i) \times (\text{Change in Time})] + [(\text{Change in Time}) \times MB_i] = \text{Sea Level Rise}$$

where

ΔD = Change in depth of the ocean due to the mass balance change over 100 years and with a 3 degree local temperature increase (sea level rise)

All other values have already been calculated or given.

Calculations:

$$\Delta D = \frac{1}{2}[(-1.06 \text{ mm}/\text{yr} - 0.38 \text{ mm}/\text{yr}) \times 100 \text{ yrs}] = -72 \text{ mm}$$

$$+ (100 \text{ yrs} \times 0.38 \text{ mm/yr}) = -34 \text{ mm} = - 3.4 \text{ cm}$$

Answer: Sea level will decrease by 3.4 cm as a result of accumulation outweighing ablation for Antarctica over 100 years and with a 3 degree increase in temperature.

END NOTES: Although this value is consistent with those calculated by the IPCC, 2001¹², the possible non-linearity of ice flow could imply that West Antarctica might be subject to some massive ice loss in the near future.¹⁸

The Results

Combining thermal expansion and mass balance contributions, we arrive at a total sea level rise of just under twenty-eight centimeters over the next hundred years given a three degree global temperature increase. After the completion of this calculation, it was a surprise to find how close this was to the current, unofficial, 2007 IPCC estimate of twenty-nine centimeters sea level rise by the year 2100.²¹ This is, to some extent, coincidental, as the IPCC models take into account factors not included in the preceding calculations.

In fact, there are two significant sources of potential error in these proceedings. The first is empirical error. Unless you personally have the knowledge, time and opportunity to utilize satellites, temperature gauges and any number of other instruments in order to measure surface areas, temperatures, ice melt and precipitation first hand, it is necessary to rely on data collected by a large community of scientists. The second significant source of potential error is the result of the simplifications intrinsic to a back-of-the-envelope calculation. Likely the greatest source of error is in our assumption that the processes examined above occur linearly. Yet this simplification of a vastly more complicated climate system still allows us to arrive at a realistic result.

From the result, it is obvious that back-of-the-envelope calculations are sufficient to adequately describe and explain the current theories regarding the estimation of sea level change due to global warming. Now, with a greater understanding, it should be possible to concentrate on the implications of what might be the most serious problem of the twenty-first century and the measures needed to counter it.

THE FINAL RESULT – ADDING UP ALL OF THE VALUES

Question: How much will the global average sea level rise over 100 years and with a 3 degree temperature increase as a result of global warming?

Given:

Sea level rise from thermal expansion = +17 cm

Sea level rise from the melting of mountain glaciers and ice caps = +9.2 cm

Sea level rise from the melting of Greenland = +5.15 cm

Sea level drop from the formation of snow and ice in the Antarctic = - 3.4 cm

Calculations:

$17 \text{ cm} + 9.2 \text{ cm} + 5.15 \text{ cm} - 3.4 \text{ cm} = 27.95 \text{ cm}$

Answer: The global average sea level will increase about 27.95 cm over 100 years and with a 3 degree temperature increase as a result of global warming.

END NOTES: According to Stephen Rahmstorf's article *A Semi-Empirical Approach to Projecting Future Sea-Level Rise*, the lowest plausible sea level rise between 1990 and 2100 (110 years) is 38 cm.²² ClimateScience.org indicates a current rate of sea level rise of 3 mm/yr, or 30 cm in 100 years (assuming this rate remains unchanged).²³ The unofficial, 2007 IPCC draft report estimates a sea level rise of 29 cm by the year 2100.²¹

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