Problem 1: A very thin ladder of length $L$ and mass $M$ leans against a vertical wall, on a horizontal floor, making an angle of $\theta$ with respect to the wall. Imagine that there is a large coefficient of friction $\mu$ at the floor so that the ladder is in static equilibrium, but assume that the wall is effectively frictionless.

(a) Draw a free-body diagram for the ladder, showing all forces acting.

(b) Using the bottom of the ladder as the axis of rotation or origin, compute all the forces and torques on the ladder such that it is in equilibrium.

(c) Why did I make the wall “effectively frictionless”?

(d) Re-solve the problem using the top of the ladder as the axis of rotation or origin. What is different in the end?

(e) At what angles $\theta$ would the ladder start to slip? If $\mu = 0.8$ (not unreasonable for rubber ladder feet on a wood floor), what is the maximum angle at which you could lean the ladder?

Problem 2: A long, thin rod of length $L$ and cross-sectional area $A$ and elastic (Young’s) modulus $E$ has mass $M$.

(a) Think of the rod as being like a Hooke’s Law spring; it can be stretched by applying a force. What is the spring constant $k$ for this spring?

(b) By dimensional analysis, can you combine $L$, $A$, $E$, and $M$ into a frequency? Do you have more than one choice? If so, which of the choices makes most sense?

(c) Look up the properties of a femur bone and compute this frequency for the femur bone.

(d) Repeat part (c) but replacing the mass $M$ of the femur bone with the mass of a typical college-age human. Hold everything else constant. That is, think of this problem as being a human mass on a femur-bone spring.

(e) Compare the frequency you got in part (c) to the dimensional analysis frequency you can obtain by combining the acceleration due to gravity $g$ with $L$ and $M$. What does this frequency represent? Is it higher or lower? Does that jive with your intuition?

Problem 3: Consider a mass $M$ on a spring of spring constant $k$, released from rest but from a distance $X$ (in the $x$-direction, which is parallel to the spring) away from the equilibrium position for the mass. Subsequently, the
mass on the spring oscillates without any loss of energy. Plot the position $x$ as a function of time, the velocity $v$ as a function of time, the acceleration $a$, the kinetic energy $K$, the spring potential energy $U$, and the total energy $E$. Make your plots in a time-aligned stack so that you can compare them.