Homework Set #2 (Due Tuesday, October 16th)

Note: All necessary information for this homework regarding solar system parameters can be found at JPL’s website\(^1\). In your writeup, you should include the information you use.

1) In class we derived the following differential equation for the orbit of a test particle around a source in general relativity,

\[
d\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + 3\frac{GM}{c^2}u^2,
\]

where \(u \equiv 1/r\) is the inverse radial position of the orbit, \(M\) is the source mass, \(h\) is the specific angular momentum of the orbit, and \(G\) is Newton’s constant.

a) Write a code that, using 4th-order explicit Runge-Kutta, solves Eq. (1).

b) Use your code for the specific case of Mercury orbiting around the Sun. Switching off the relativistic term in Eq. (1), check that your code produces a closed orbit and compare it to the exact Newtonian result. Make a plot of this comparison for a few different choices of the timestep in your integrator.

c) Use your code for the relativistic case and compare the result you get for the perihelion precession with that of the perturbative result we obtained in class. Hint: at the beginning, you may want to artificially increase the precession to make easier its detection by amplifying arbitrarily the last term in Eq. (1). Also, in order to avoid making the timestep very small (which makes it slow) to find the perihelion, you may want to fit a quadratic to the three points closest to perihelion to better find the angle at perihelion.

2) Suppose the gravitational Newtonian force is changed to

\[
F = \frac{GMm}{r^2} \times \left(\frac{r_0}{r}\right)^\delta
\]

a) Find the equation of the orbit for this force law.

b) By using the same perturbation method we did for GR in class, find the shift in perihelion.

c) Using your Runge-Kutta solver, compute the perihelion shift for \(\delta = 0.05\) and \(r_0 = h^2/GM\), where \(h\) is the angular momentum per unit mass. Compare with your perturbative answer in b).

\(^1\)http://ssd.jpl.nasa.gov