Most successful inflationary models are based on one or more scalar fields, driving inflation. (The simplest one - "slow roll" scenario involves a massive scalar field, slowly changing with time.) These models have been very successful in explaining many properties of the observable universe, however the question of identifying inflator with any known physical scalar field is still open. (In fact, the only known fundamental scalar field - Higgs has already been ruled out as a candidate for inflator), therefore it is natural to consider the possibility of higher spin fields to drive inflation, for which such a connection may be made more easily.

1. Introduction

Two main reasons why scalar field condensates have been more successful candidates for inflation, than vector field condensates, are a) natural homogeneity and isotropy of scalar fields and b) their ability to imitate slow roll regime, which has been a problem for vector fields. (For example, in one of the vector inflation models, based on a potential $V(A^2A^2)$, it doesn't change much during inflation, whereas $A^2A^2$ run over exponentially large range of values). Both of these obstacles can be overcome by considering 3 mutually perpendicular or some large number $N$ of randomly oriented vector field
2. THE MODEL

Let us consider a theory with a massive vector field, nonminimally coupled to gravity:

\[ S = \int d^4x \sqrt{-g} \left( -\frac{R}{16\pi G} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (m^2 + \frac{R}{6}) A_\mu A^\mu \right) \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [ A_\mu, A_\nu ] \]

and we use Planck units \((G = 1)\).

The variation of action with respect to \(A_\mu\) yields the following equs of motion:

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} F^{\mu\nu} \right) + (m^2 + \frac{R}{6}) A_\nu = 0. \quad (\star) \]

As the background metric, we consider spatially flat FRW:

\[ ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j. \]

\((\star)\) leads to the following equations:

\[ \frac{1}{a^2} \dot{A}_0 + (m^2 + \frac{R}{6}) a_0 + \frac{1}{a} \dot{a} \dot{A}_0 = 0. \quad (1) \]

\[ \ddot{A}_i + \frac{3}{a} \dot{a} \dot{A}_i - \frac{1}{a^2} \ddot{A}_i + (m^2 + \frac{R}{6}) A_i - \frac{1}{a} \dot{a} A_0 + \frac{1}{a^2} \dot{a} (\dot{A}_0 A_i) = 0. \quad (2) \]

Where \(\ddot{A}_i = \frac{d}{dt} \frac{d}{dx_i}\). Let us also introduce \(B_i = \dot{A}_i\). Considering the quasi-homogenous vector field \((\dot{a} A_0 = 0)\), we immediately infer from \((1)\) that \(A_0 = 0\). Eq. \((2)\), rewritten in terms of \(B_i\), gives:

\[ B_i + 3HB_i + m^2 B_i = 0. \quad (3) \]

\(H = \frac{\dot{a}}{a}\), which looks exactly the same as the equation for the scalar field, minimally coupled to gravity. When the Hubble constant is larger than mass of the field, \(H > M\), fields are "frozen". Let us now check, if the energy-momentum tensor of the field condensate is consistent with the inferred homogeneity and isotropy of the metric.
Varying the action with respect to $g_{\mu\nu}$, we obtain:

\[ T^\alpha_{\beta} = \frac{1}{4} F^\alpha_{\gamma\delta} F^\beta_{\gamma\delta} - \frac{1}{2} R F_{\alpha\beta} + \frac{1}{2} (m^2 + \frac{\kappa}{6}) A^{*} A_{\alpha} - \frac{1}{2} m^2 A^{*} A_{\alpha} A^{*} A_{\beta} + \]

\[ + \frac{1}{6} (\kappa Q_{\beta} - \frac{1}{2} S^0_{\beta} R) A^{*} A_{\alpha} + \frac{1}{6} (\kappa S_{\beta} - \frac{1}{2} A_{\beta} A^{*} A^{*} A_{\alpha}) A^{*} A_{\alpha}. \]

For an homogeneous vector field in a flat Friedmann universe, we have:

\[ T^0_{\alpha} = \frac{1}{2} (B^2_k + m^2 B^2_k) \]

\[ T^i_{\alpha} = [\frac{-5}{6} (B^2_k - m^2 B^2_k) - \frac{2}{3} H B^2_k B_k - \]

The spatial part of the E.M. -\[ \frac{1}{3} (H + 3 H^2) B^2_k S^i_{\alpha} + B^i_{\beta} B^j_{\gamma} + H (B^i_{\beta} B^j_{\gamma} + B^j_{\beta} B^i_{\gamma}) + \]

tensor has non-diagonal \[ + (H + 3 H^2 - m^2) B^i_{\gamma} B^j_{\delta}. \]

components of the same order, as the on-diagonal ones, and these is an isotropic universe, filled with homogeneous vector field condensate is not consistent with Einstein's equations, which is not surprising, because of the preferred direction of vector field.

3. \textbf{INFLATION}.

The situation can be cured by introducing a triplet of mutually orthogonal vector fields $B^i_{(a)}$ with the same magnitude $1B_1$ each:

\[ \sum B^i_{(a)} B^i_{(b)} = 1B_1^2 \delta^{ab} \Rightarrow \sum B^i_{(a)} B^j_{(b)} = 1B_1^2 \delta^{ij}. \]

Using these relations, we find that the total E.M. Tensor becomes equal to:

\[ T^0_{\alpha} = \frac{1}{2} (B^2_k + m^2 B^2_k) ; T^i_{\alpha} = -p \delta^i_{\alpha} = -\frac{1}{2} (B^2_k - m^2 B^2_k) S^i_{\alpha}. \]

where $B_k$ are components of any field from the triplet which satisfy:

\[ \dot{B}^i_{\alpha} + 3H B^i_{\alpha} + m^2 B^i_{\alpha} = 0. \]

where $H^2 = \frac{4\pi}{3} (B^2_k + m^2 B^2_k)$

These equations are precisely the same as for the massive scalar field and the consequences are also the same, in particular during $B_k < m^2 B_k^2$ stage, we have inflation.
Another way to resolve the issue of anisotropy is to consider a large number of randomly oriented vector fields. Let us consider \( N \) fields of equal mass \( m \) assuming they all have the same magnitude \( B \) initially. The components of these fields satisfy (3) and their total contribution to \( T_0^0 \) is:

\[
T_0^0 = E = \frac{N}{2} (B_x^2 + m^2 B_z^2)
\]

To get an estimate of the spatial components of the energy-momentum tensor we note:

\[
\sum_{\alpha = 1} B_\alpha \cdot B_\beta = \frac{N}{6} B^2 \delta_{ij} + \mathcal{O}(1/N) B^2.
\]

where \( B^2 = B_x^2 \). Corrections, proportional to \( 1/N \) are due to stochastic random distribution of directions of the fields and they don't vanish for \( i \neq j \). During inflation, the typical value of off-diagonal E.M. tensor components is of order \( H^2/N B^2 \). Isotropic inflationary solution is self-consistent only if these components are less than diagonal ones: \( T_{ij} \propto H^2 = \frac{1}{N} \). On the other hand, we have slow-roll regime only if \( H > m \). In the inflationary stage \( H^2 = \frac{8\pi}{3} \sum_i \frac{1}{2} m^2 B_i^2 \), we obtain:

\[
\frac{1}{N} > B > \frac{1}{N}
\]

Let us estimate the number of e-folds during the inflationary stage: \( \eta \sim -\frac{m^2 B}{3H} \) and we obtain:

\[
\frac{\Delta i}{\Delta i} \propto \exp(2\pi NB_i h) \text{ where } B_i = \text{ initial value of the vector fields. For } B_i = \frac{1}{\sqrt{N}}, \text{ we find - number of e-folds } \sim 2\pi \sqrt{N} = \sim \text{ several hundred is enough to explain the observed homogeneity. At the end of inflation there still survive the off diagonal spatial components of E.M. tensor which induce the anisotropy of relative magnitude } \frac{1}{\sqrt{N}}.
\]
Conclusions:
This model of inflation discussed a possibility of 3 mutually perpendicular or some number N of randomly oriented vector fields to drive inflation. In the latter case, the anisotropy of relative order $\frac{1}{N}$ survives after inflation. The very light fields remain frozen until today and thus serve as the observed dark energy which can be even anisotropic.