Experimental Constraints on Modified Gravity
at low scales.

Many theories give rise to modifications of
gravity at either large or small
distinct scales.

To clarify, when I speak of modified
gravity, I mean true modifications to
gravitational theory or apparent
modifications due to new forces mediated
by unknown particles (e.g. fifth force).

Theories of extra dimensions and more general
theories that contain additional light particles
also give rise to modifications of gravity
at small scales (<1 mm). Examples include
ADD (extra dim <1 mm)
RSI (1 small warped extra dim)
RSII (1 infinite extra dim)
string theories with light moduli
theories of axions
theories of SUSY w/ hidden sectors

Long distance (>1 mm) modifications can occur
in theories in which long distance gravity
is higher dimensional, so called theories of
the "quasiglobalization of gravity" in this
case 1/r^2 only holds within some range.
such theories may solve the cosmological constant problem through a change in gravity at the Hubble scale. 

we can characterize the nature of the modification in a Yukawa potential

\[ V(r) = -G_{\text{w}} m_1 m_2 \left( 1 \pm e^{-r/r_0} \right) \]

where

- \( \pm \): strength of modification relative to gravity
- \( r_0 \): range of the modification

\( r_0 = h/m_c \) the Compton wavelength of particle mediating the interaction

+ sign corresponds to a repulsive potential \( (\text{exchange of a vector particle}) \)

- sign corresponds to an attractive potential \( (\text{exchange of a scalar or } \sigma=2 \text{ tensor particle}) \)

the corresponding force is given by

\[ F(r) = -G_{\text{w}} \frac{m_1 m_2}{r^2} \left[ 1 \pm e^{-r/r_0} \right] \left( 1 + \frac{r}{r_0} \right)^2 \]

\( \sigma \) note: other forms for \( V(r) \) are possible corresponding to long-range forces \( \sigma \approx \frac{1}{4}\pi \)
From "Improved Constraints on km-Newtonian forces@10m" Geraci et al.

MICRO CANTILEVER EXP (ca. to Newton (10^-11 N) sensitivity)

COMPONENTS (show Fig 1)

- Silicon cantilever: 250 x 50 x 3 mm^3, k = 0.0002 N/m, resonant frequency = 300 Hz

- Test mass - gold rectangular prism, m = 1.5 mg, 54 x 54 x 27 mm^3

- Driving mass (source) - provides density modulations through 100 nm wide alternating bars of gold and silicon, oscillates 1 length of cantilever at vertical separations down to 1 nm

- Silicon-nitride shield between cantilever, drive mass, attenuates electrostatic & Casimir forces (backgrounds)

- This apparatus is mounted in side a cryostat which is hung from ceiling by springs - the operating temp of the experiment is T = 11-13 K, a magnetic shield surrounds the apparatus

- Interferometer used to measure the displacement of the cantilever from its equilibrium position (interferometer measures interference of light reflected off of test mass w/ interfer. of light reflected off end of fiber)
- Drive mass oscillates at \( \frac{1}{3} \) the resonant frequency of the cantilever (\( \approx 100 \text{ Hz} \)) with an amplitude of \( \approx 120 \mu \text{m} \) above its equilibrium position.

- The gravitational coupling between the masses creates a force \( (2) \) harmonics of the dual frequency, including at \( 3f_{\text{drive}} \) which is the resonant frequency.

- In this way the motion of the test mass is at a definite phase with drive mass motion and at a harmonic of the drive frequency.

- Force is measured as a function of the equilibrium y-position between the drive & test masses.

**What is measured?**

Position \( X \) and positions are held constant while drive mass is allowed to oscillate about its equilibrium position \( y \).

The interferometer puts out a time dependent signal, the Fourier Transform of this signal.
is taken and the 3rd harmonic of the trial signal is isolated. The FT of the intensity is converted to an amplitude of the cantilever motion which is in turn converted to a force on the cantilever (3rd harmonic of AC force).

INTERFEROMETER $\rightarrow$ FT. of IS/ 3rd HARM. $\rightarrow$ FT. of SOURCE/3rd H

from the FT, the amplitude and phase, or equivalently the real $\tilde{F}$ and imaginary parts, of the force are found.

Note: apparatus runs for time $t_0$. This interval is broken up into smaller intervals to for analysis of data (separated so no correlation),
the number of records/data points per run is then $t_0 \div t = N$

the $N$ measurements of $F_\tilde{R}$ and $F_\tilde{I}$ are averaged.
Signal (show Fig 2)
- Here we see expected Yukawa signal — amplitude and phase — for $\omega = 1000$, $d = 15$ um.
- Of the 3rd harmonic of the drive/resonance frequency of the cantilever.

- We see that 3rd harmonic of force vanishes when gold or silver bar is centered under the test mass during oscillation.
- Phase change every 100 um.

Measured Signal (show Fig 5)
- 17 data points corresponding to 24-120 min of averaged data ($N = ?$)

- Y-extent of data should include three Yukawa phase changes, each separated by 100 um — instead we see a nearly constant phase — (note this is not thermal noise — this would result in a randomly distributed phase centered on zero see page 7).

Data was taken for $d$ values of $d = 4, 6, 10, 18, 34, 66$ um.
Error

Error in a bound on or discovery of a Yukawa-type correction to Newtonian gravity can be sorted into two types:

1) Error associated w/ measurement of force and (2) error associated w/ calculation of expected force signal

Errors in measurement of force

- Statistical error due to thermal noise
  
  Contributed > 90% to 10-error bars in Fig 5 — this decreases with \( \sqrt{N} \)
  
- Systematic errors — the uncertainty in the position of the fiber contributes an uncertainty of 12% in the force measurement. Out of a total systematic error of 13% at 10...

Error in calculation of force is due to

- Uncertainty in vertical separation \( z \)
- Tilt of driving mass
- Rimorph amplitude
- Geometry & density of masses
- y position of data points
Fitting data to theory

- to obtain a function $\alpha(r)$ for a Yukawa force that best fits the data, they considered a least-squares fit of the real and imaginary parts of the data to a set of calculated forces in the Monte-Carlo simulation (given $r$, separation $r$, hits - average values, means assuming Gaussian dist)

Solving Parameters

- $\lambda_0$ offset variable to account for uncertainty of $y$ equilibrium position of drive mass
- real $\theta$, imaginary parts of a constant offset force (to account for backgrounds that don't depend on $y$-equilibrium position of drive mass)

A complicated analytic process takes place resulting in a histogram of 3,210,000 best-fit $\alpha$'s for each of the six $\delta$'s

(Show Table 5)

Here we see the mean values of $\alpha$ and the 95% confidence interval (mean value represents the most likely value)
the results represent an improvement of the constraints on Yukawa forces by a half order of magnitude over previously published work

\[ \text{show Fig.9} \]

Note: Modifications to gravity with strength equal to that of gravity have been ruled out down to \( \lambda \leq 15 \mu m \)

Lamoreaux - Casimir force exp
UV - low freq. torsion oscillator
VC - high freq. torsion oscillator

* these measurements represent the best bound in the range 5-15 \( \mu m \) w/ a 95% confidence exclusion of forces w/ \( |x| > 14,000 \) at \( d = 10 \mu m \)

Future design contours to allow a larger test mass resulting in an improvement of 1-2 orders of magnitude