1) Given the vector field \( \mathbf{G} = y\hat{e}_x - x\hat{e}_y + z\hat{e}_z \), verify Stokes’ theorem for the hemispherical surface \( x^2 + y^2 + z^2 = a^2 \), with \( z \geq 0 \).

2) Show that the following ODE is exact,

\[
y(2x^2y^2 + 1)y' + x(y^4 + 1) = 0
\]  

Find the solution that satisfies the boundary condition \( y(0) = 1 \).

3) Find a power series solution to the following ODE,

\[
(1 - x^2)y'' - 2xy' + n(n + 1)y = 0
\]

Use the recurrence relations that you find to write down explicitly the first three terms of the solution.

4) Find the steady state temperature distribution inside and outside of a sphere of radius \( a \) that is held to temperature \( T = T_0 \sin^2(\theta) \).

5) Find the steady state temperature distribution inside a cylinder of height \( h \) that is held to \( T = 0 \) at the bottom and walls, and \( T = T_0 J_2(\alpha_{22} \rho/a) \sin(2\phi) \) at the top, where \( \alpha_{22} \) is the second root of the Bessel function \( J_2(x) \), and \( a \) is the radius of the cylinder.

6) Find the electric potential inside of a square two dimensional plate of side \( L \), that is held to \( \Phi = 0 \) at the bottom and left sides, and \( \Phi = V_0 \) at the top and right sides.