CMB Anisotropies
Useful Reads:

- Hu & Dodelson, ARAA, 2002, “Cosmic Microwave Background Anisotropies”
- Wayne Hu’s website:
  - [http://background.uchicago.edu/index.html](http://background.uchicago.edu/index.html)
- Max Tegmark’s Website:
  - [http://space.mit.edu/home/tegmark/movies.html](http://space.mit.edu/home/tegmark/movies.html)
Spherical Harmonics

\[ Y_{\ell m}(\theta, \phi) \propto e^{i m \phi} P_{\ell}^{m}(\cos \theta) \]

associated Legendre polynomials

Pattern of \( Y_{\ell m} \): grid of \( m \) great circles passing through the poles and \( l \)-\( m \) circles of equal latitude; zero at these boundary lines, changing signs across them.
Spherical Harmonics Expansion of the Temperature Fluctuation Field

\[
\frac{\Delta T(\theta, \phi)}{\bar{T}} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi)
\]

\[
\Delta T(\theta, \phi) \equiv T(\theta, \phi) - \bar{T}
\]

Variance

\[
\left\langle \left( \frac{\Delta T(\theta, \phi)}{\bar{T}} \right)^2 \right\rangle_\Omega = \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} C_l
\]

\[
C_l = \frac{1}{2l + 1} \sum_{m=-l}^{l} \langle |a_{lm}|^2 \rangle
\]

in analogy to \( P(k) \)

usually plot

\[
\Delta_T^2 \equiv \frac{l(l+1)}{2\pi} C_l T^2
\]

\[
\Delta^2 \equiv 4\pi k^3 P(k)
\]

\[
l \leftrightarrow k
\]

\[
\sim l^2 C_l \leftrightarrow k^3 P(k)
\]

for large \( l \), fluctuation power per logarithmic \( l \)

fluctuation power per logarithmic \( k \)

2D \quad \begin{align*}
C_l & \leftrightarrow P(k) \\
& \sim l^2 C_l \leftrightarrow k^3 P(k)
\end{align*}

3D
Another reason to plot $l(l+1)C_l$

For $n_s=1$ scale-invariant spectra (Harrison-Zel’dovich spectra)

$$\frac{l(l + 1)}{2\pi} C_l = \text{const}$$

- As a rule of thumb, the angular “wavenumber” $l$ corresponds to an angular scale
  $$\theta \sim \frac{\pi}{l} \text{radian} \sim \frac{180^\circ}{l}$$

- For flat $\Lambda$CDM cosmology, the angular size corresponding to the horizon size at decoupling ($z_{\text{dec}} \sim 1100$) is
  $$\theta_H = \frac{c}{H(z_{\text{dec}})} \frac{D_A(z_{\text{dec}})}{D_A(z_{\text{dec}})} \sim 1^\circ \quad l \sim 200$$
Key Points

• before decoupling, photon-baryon fluid

• after decoupling, free-streaming of photons

• snapshot of perturbations at the time of decoupling

• a wealth of cosmological information
Primary Anisotropies
• Anisotropies that arise at the time of recombination and subsequent free-streaming of photons.
  • temperature variations (i.e., photon density)
  • gravitational redshift of photons
  • time-varying gravitational potential
  • Doppler shifts from fluid motion at last scattering.
  • gravitational waves (aka “tensor fluctuations”)

Secondary Anisotropies
• Arise when photons are rescattered at lower redshifts.
  • Doppler shifts at secondary scattering.
  • Thermal Sunyaev-Zeldovich effect.
Foregrounds

- Contaminants to the microwave fluctuations.
- Discrete radio sources.
- Thermal emission by galactic dust.
- Radiation from spinning dust.
- Galactic synchrotron radiation.
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Also...

• CMB photons scatter off moving electrons, thus fluctuations expected to by polarized at \(~10\%\) level.
• Gravitational lensing has a weak but detectable (detected) effect on anisotropies.
Close-coupling approximation: treating photons and baryons and single ideal fluid prior to recombination

Inside the horizon, an overdensity will start to collapse under its own self-gravity. Crossing time for pressure gradient to build up is:

Photon pressure causes perturbations to oscillate in time, so temperature fluctuation oscillates from positive to negative and back.

\[ c_s = \frac{c}{\sqrt{3(1 + R)}} \]
\[ R = \frac{3\rho_b}{4\rho_\gamma} \]

\[ t_{cross} = \left( k c_s / \pi \right)^{-1} \]

\[ t_{cross} \lesssim t_* \]
Key Features of CMB power spectrum

• roughly flat at $l=2-30$ (modulo the low quadrupole $[l=2]$, but cosmic variance high)

• rises to strong peak at $l=220$

• shows clear evidence for 2nd and 3rd peaks, with lower amplitude.
Angular Size of Hubble Radius at Decoupling ($z_{\text{dec}} \sim 1100$)

$$\frac{c}{H(z_{\text{dec}})} \sim 1^\circ$$

scales larger than the 1st peak -- super-horizon

**Sachs-Wolfe Effect**

- fluctuations in the energy density of photons ($\rho_{\gamma} \propto T^4$)
  +
- fluctuations in the gravitational potential (redshifts of photons when climbing out of a potential well)
  =
- fluctuations in observed photon temperature
Let’s focus on regions of potential well (overdense, in DM, baryons, photons, ...; adiabatic initial condition from inflation)

**Sachs-Wolfe Effect**
- fluctuations in the energy density of photons ($\rho_\gamma \propto T^4$)
- fluctuations in the gravitational potential (redshifts of photons when climbing out of a potential well)

\[ \frac{\Delta T}{T} = -\frac{2}{3} \Psi \] (super-horizon, GR result)

In the potential well (over-dense region, perturbed potential $\Psi < 0$), photons have a higher temperature.

These photons need to climb out of the potential well (after recombination) to be observed

\[ \left. \frac{\Delta T}{T} \right|_{\text{obs}} = \Theta + \Psi = \frac{1}{3} \Psi \]

Gravitational redshift wins! Photons are colder in overdense regions.
CMB Angular Power Spectrum

\[ \frac{l(l+1)C_l}{2\pi} \sim 180 \]

Super-horizon Scale Perturbations
(Initial Conditions)

The fluctuation amplitude at the time of exiting the inflation horizon is roughly

\[ \delta_H \sim -2\Psi = -6 \left| \frac{\Delta T}{T} \right|_{\text{obs}} \]

(N.B. a more accurate calculation shows that the rms fluctuation has the factor 5 instead of 6)

The Sachs-Wolfe part of the spectrum \((\Delta T \sim 30 \mu K)\) tells us that

\[ \delta_H \sim 5 \times 10^{-5} \]
The physical scale of the 1st acoustic peak corresponds to the sound horizon at recombination.

The angular scale of the 1st acoustic peak depends on the curvature of the universe.

The fact that the 1st peak appears at $l \approx 200$ (plus $H_0 \approx 70$ km/s/Mpc) tells us that the universe is spatially flat.