Homework 2

Problem 1.
Centripetal acceleration is given by \( a = \frac{v^2}{R} = \omega^2 R \), where \( v \) is velocity, \( R \) is radius of rotation, \( \omega = \frac{v}{R} \) is angular velocity defined as change of angle in unit time (very useful for rotation problems). The Earth makes one rotation (angle \( 2\pi \)) in 24 hours, that means \( \omega = \frac{2\pi}{24} \) hours\(^{-1} = \frac{2\pi}{24}/3600 \) s\(^{-1} \). Radius of the Earth is \( R = 6370 \ km \approx 6.4 \times 10^5 \ m \). So, the centripetal acceleration at equator is \( a = \omega^2 R \approx 0.034 \ m/s^2 = 3.4 \times 10^{-3} g \), where \( g = 9.8 \ m/s^2 \).

Problem 2.
see graphs.

Problem 3.
(i) Constant acceleration \( x_0 = 0 \) and \( v_0 = 0 \), so \( x = at^2/2 \). (ii) Constant speed \( v_a = at_a \). Total distance \( x_f = at_a^2/2 + v_a(t_f - t_a) = at_a^2/2 + at_a(t_f - t_a) = at_a(t_f - t_a) - at_a^2/2 \). There are 2 cases: (a) \( t_a < t_{1/2} \), (b) \( t_a > t_{1/2} \).

(a) \( t_a < t_{1/2} \) - half of the distance is \( x_f^2/2 = at_a^2/2 + v_a(t_{1/2} - t_a) \) and for total \( x_f = at_a^2/2 + v_a(t_f - t_a) \). Substituting \( v_a = at_a \) we get

\[
\begin{align*}
  x_f &= at_a(t_f - t_a) - at_a^2/2 = at_a(t_f - t_a)/2, \\
  x_f/2 &= at_a(t_{1/2} - t_a)/2 = at_a(t_{1/2} - t_a)/2, \\
  x_f/2 &= at_a(t_{1/2} - t_a)/2 = at_a(t_{1/2} - t_a)/2,
\end{align*}
\]

now let's divide one equation by the other

\[
2 = \frac{at_a(t_f - t_a/2)}{at_a(t_{1/2} - t_a/2)} = \frac{t_f - t_a/2}{t_{1/2} - t_a/2}.
\]

From here we get for time of accelerated motion \( t_a = 2(2t_{1/2} - t_f) = 1.84 \) s and correspondingly for acceleration \( a = 5.9 \ m/s^2 \).

(b) \( t_a > t_{1/2} \) - half of the distance is \( x_f^2/2 = at_a^2/2 \) and for total \( x_f = at_a^2/2 + v_a(t_f - t_a) \). From the first equation we get acceleration. From the second we find time \( t_a \) (by solving quadratic equation), but both solution are out of range: the first one is less then \( t_{1/2} \), the second is bigger then \( t_f \). One could see this by making graph \( v \) vs. \( t \).
Figure 1: Problem 2.