Homework 5.

Problem 1.
Since the string is continuous and the pulleys are frictionless, the tension is the same all over the string. If we’ll assume that acceleration of mass $m_2$ is $a$ down than acceleration of $m_1$ is $a/2$ up (move $m_2$ by $\Delta x_2$ down, then $m_1$ will move by $\Delta x_1 = -\Delta x_2/2$, minus stays for up). Thus we can write equation of motion for both masses

$$m_1a/2 = 2T - m_1g, \quad m_2a = m_2g - T.$$ We get for accelerations and tension:

$$a_2 = a = \frac{2m_2 - m_1}{2m_2 + m_1/2}g, \quad a_1 = -\frac{a}{2}, \quad T = \frac{3}{2} \frac{m_1m_2}{2m_2 + m_1/2}g.$$  

(Check the limits for consistency $m_1 \rightarrow 0, \; m_2 \rightarrow 0, \; m_1 \rightarrow 2m_2$.)

Problem 2.
1 gal $\approx 3.5$ liters, since gas is a bit less dense than water, the mass of 1 gal of gas is about 3 kg. Atomic weight of gas is about 100 it means that Avogadro’s number $N_A \approx 6 \times 10^{23}$ of gas molecules make 100 gram. Thus, there are $N = 30N_A$ molecules in 3 kg of gas. If the burning energy of a single molecule is $5 \times 10^{-19}$ J than 3 kg of gas produce energy $E \approx 30 \times 6 \times 10^{23} \times 5 \times 10^{-19} J \approx 10^7 J$, which is enough to make 25 miles $\approx 4 \times 10^4 km$ at speed 55 miles/h.

The work done by friction forces equals to useful work done by the engine $W = F_f l = .25E$, where $l$ is the path of the car. Thus, the total "friction" force acting on the car at 55 miles/h is about $F_f \approx 50 N$.

Power is work done in a unit of time $P = .25E/\Delta t$, where $\Delta t$ is time in which car makes 25 miles. $P \approx .25E/25\text{ min} \approx 1.7 \times 10^3 J/s = 1.7 kWatt \approx 2.5 hp$ it is about a percent of power for typical car, which is pretty reasonable.

Problem 3.
Let’s see what forces are acting on a car: force of gravity down $mg$, reaction force normal to the to the road $N$ and friction force tangent to the road $F_f$ and we know that $|F_f| \leq \mu N$ where $\mu$ is friction coefficient. Road have been optimize, that means reducing force of friction to zero. Thus centripetal force, forces car to make turn, is produced just by the projection of force of reaction to the center of curvature $F_c = mv_0^2/R = N \sin \theta$. At the same time, reaction force should compensate gravity $mg = N \cos \theta$. From here we find slope $\theta$: $\tan \theta = \frac{v_0^2}{Rg}$ or $\theta \approx 22^\circ$, where $v_0 = 55 mile/h$ is optimization speed.

Now we know $\theta$, let’s calculate the maximal safe speed. Centripetal force is produced by projections of force of reaction and force of friction $F_c = mv^2/R = N \sin \theta + F_f \cos \theta$. 

1
Since we are interested in the maximal speed we are using maximal force of friction \( F_f = \mu N \). Thus we get \( mv^2/R = N \sin \theta + \mu N \cos \theta \). Taking into account that now \( mg = N \cos \theta - F_f \sin \theta \) we get for maximal safe speed

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v = \sqrt{\frac{gRv_0^2 + \mu gR}{gR - \mu v_0^2}} \approx 40.8 \text{ m/s} \approx 92 \text{ mile/h}.
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