Homework 6.

Problem 1.
- Bullet fired from a rifle: speed $v \approx 1000 \text{ m/s}$, mass $m \approx 10 \text{ g} = .01 \text{ kg}$, momentum $p = mv \approx 10 \text{ kg m/s}$, energy $E = \frac{mv^2}{2} \approx 5000 \text{ J}$.
- A soccer ball kicked hard: speed $v \approx 30 \text{ m/s}$ (soccer field size is 100m, it takes about 3s for a ball to get a bit farther then middle of the field after goalkeeper kicks it), mass $m \approx 500 \text{ g} = .5 \text{ kg}$, momentum $p = mv \approx 15 \text{ kg m/s}$, energy $E = \frac{mv^2}{2} \approx 250 \text{ J}$.
- A three year old on tricycle: speed $v \approx .5 \text{ m/s}$, mass $m \approx 20 \text{ kg}$, momentum $p = mv \approx 10 \text{ kg m/s}$, energy $E = \frac{mv^2}{2} \approx 2.5 \text{ J}$.

Problem 2.
Masses of the blocks $m_1 = 5 \text{ kg}$ and $m_2 = 2 \text{ kg}$, corresponding initial velocities $v_1 = 7 \text{ m/s}$ and $v_2 = -2 \text{ m/s}$. The collision is elastic that means the energy is conserved. Let’s velocities after collision be $u_1$ and $u_2$ then

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2, \quad \text{‐ conservation of momentum}$$

$$\frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}, \quad \text{‐ conservation of energy.}$$

Let’s rewrite above formulas by taking terms with $m_1$ to the left and with $m_2$ to the right:

$$m_1(v_1 - u_1) = m_2(u_2 - v_2), \quad \frac{m_1}{2}(v_1^2 - u_1^2) = \frac{m_2}{2}(u_2^2 - v_2^2)$$

Dividing one equation by the other we get following system

$$m_1(v_1 - u_1) = m_2(u_2 - v_2), \quad v_1 + u_1 = u_2 + v_2.$$

It is now trivial to solve

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2 = \frac{6}{7} \text{ m/s}, \quad u_2 = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_2 + m_1}v_2 = \frac{10}{7} \text{ m/s}$$
Problem 3.

For the first block, which is sliding down, we have projection of force of gravity \( Mg \sin \theta \) bigger than the maximal friction force \( f_k = \mu_k N = \mu_k Mg \cos \theta \). Thus we get acceleration \( Ma = Mg \sin \theta - \mu_k Mg \cos \theta \), and correspondingly for acceleration itself \( a = g(\sin \theta - \mu_k \cos \theta) \geq 0 \) (we should have \( \mu_k \leq \tan \theta \)).

For the second block, which is static, we have projection of force of gravity \( Mg \sin \theta \) is exactly compensated by friction force \( f_s \leq \mu_s N = \mu_k Mg \cos \theta \). In other words \( Mg \sin \theta - f_s = 0 \), or \( f_s = Mg \sin \theta \) (we should have \( \mu_s \geq \tan \theta \)).

Problem 4.

If \( m_2 \) falls by a distance \( \Delta h \), \( m_1 \) would rise by \( \Delta h/2 \). Consequently, the change of potential energy is \( \Delta U = m_1g\Delta h/2 - m_2g\Delta h = (m_1/2 - m_2)g\Delta h \). If \( m_1/m_2 = 2 \) than \( \Delta U = 0 \), so kinetic energy should not change as well. The acceleration for this ratio is zero.