NYU Engineering Physics 1—Problem set 11

Due Thursday 2004 April 15 by 4:30pm at Irene Port’s office.

Problem 1:  A mass of 10 kg hangs on a light string of length \( l = 9.8 \text{ m} \) and swings as a pendulum in gravity. The pendulum is released from rest at time \( t = 0 \) with an initial angle to the vertical of \( \theta_i = 0.01 \text{ rad} \).

(a) Compute the torque \( \tau \) on the pendulum at time \( t = 0 \). Compute approximate values for the angle of the pendulum \( \theta(0.1 \text{ s}) \) and its angular velocity \( \omega \equiv d\theta/dt \) at time \( t = 0.1 \text{ s} \) by computing the gravitational torque on the pendulum at time \( t = 0 \) and using that to increment the angular velocity from rest. Treat the torque and velocity as not changing by much within the interval.

(b) Compute approximate values for the angle of the pendulum \( \theta(0.2 \text{ s}) \) and its angular velocity \( \omega(0.2 \text{ s}) \) at time \( t = 0.2 \text{ s} \) by computing starting with what you computed in part (a), and again approximating the torque and velocity as not changing much within the interval. Do not use your calculator!

(c) Continue the above, computing, approximately, the torque, the angle and angular velocity on a grid of times 0.1, 0.2, 0.3, \ldots, 2.0 \text{ s}. You may use a calculator.

(d) Obtain an approximate value for the time \( t_{eq} \) at which the pendulum gets to its equilibrium angle (\( \theta = 0 \)) and the angular speed \( \omega_{\text{max}} \) it has at that point, by interpolating to the point at which your approximation crosses zero. Explain any discrepancy you get from the true value. Suggest two ways you could you have improved your calculation.

Does this seem familiar?

Problem 2:  A mass on a spring oscillates in the \( x \)-direction around \( x = 0 \) with a period of \( T = 10 \text{ s} \) with no damping force or friction. If at time \( t = 0 \) the mass has displacement \( x = +0.1 \text{ m} \) and \( x \)-direction speed \( v_x = -0.3 \text{ m/s} \), compute the parameters \( A, B, \) and \( \omega_0 \) in the parameterization of its trajectory

\[
x(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t) \tag{1}
\]

Compute the parameters \( x_0, \phi, \) and \( \omega_0 \) in the parameterization

\[
x(t) = x_0 \cos(\omega_0 t + \phi) \tag{2}
\]
Note that these two expressions for \( x(t) \) are *mathematically equivalent*. They both completely express the general solution to the differential equation

\[
\frac{d^2x}{dt^2} + \omega_0 x = 0.
\]  

(3)

**Problem 3:** Show, by explicitly taking derivatives, that the function

\[
x(t) = A e^{-\gamma t/2} \cos \omega_1 t,
\]

(4)

where \( \omega_1 \equiv \sqrt{\omega_0^2 - \gamma^2/4} \), is a solution to the differential equation

\[
\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0
\]

(5)

**Problem 4—optional (not for credit):** A child of mass \( m \) sits exactly on top of a hemispherical mound of frictionless ice of radius \( R \). If the child is displaced a tiny (ie, small relative to \( R \)) horizontal distance \( x \) from the top of the mound of ice, what is the \( x \)-component \( F_x \) of the net force on the child? Write down the differential equation relating the \( x(t) \) to its second derivative (with respect to time). What functions \( x(t) \) solve your equation? Try to be as general as possible. *Hint: Try exponentials!*.