Problem 1

Estimate the capacitance, in \( \mu F \), of a storm cloud over Lake Ontario. Treat the system as a parallel-plate capacitor with the area of the lake and a separation of 1000 ft. How much energy is stored in the capacitor when lightning strikes? To compute this, you will have to use the potential difference associated with the breakdown of air over a distance of 1000 ft. Give your answer in J and in kilotons of TNT equivalent, where 1 kiloton is about \( 4 \times 10^{12} \) J.

Problem 2

Compute the capacitance of a cylindrical capacitor in which the inner plate is a cylinder of radius \( a \) and the outer plate is a cylinder of radius \( b > a \), both of length \( l \). Compute the capacitance by (a) explicitly computing the electric field magnitude \( E(r) \) as a function of cylindrical radius \( r \) for a charges of \( +Q \) and \( -Q \) on the plates, and (b) explicitly integrating the field to get the potential difference \( V \). Take the limit in which \( (b-a) \ll a \), and show that the capacitance you get is the same as a “rolled-up” parallel-plate capacitor.

Problem 3

Integrate the energy density \( \epsilon_0 |\vec{E}|^2/2 \) over the volume in the cylindrical capacitor of Problem 2, and show that the total energy stored in the field is equal to \( Q^2/(2C) \). Do not assume that \( (b-a) \ll a \). This problem involves setting up a non-trivial integral; show all your steps (because setting up the integral is much more important than integrating it). What is the energy, in J, in a piece of coaxial cable (like the one connected to your television) of length 1 m, inner radius 1 mm, and outer radius 2 mm, charged up to 120 V?