Big Bang Nucleosynthesis

B_{\gamma} = Z m_p + (A-Z)m_n

Deuterium \( \text{^2H} \)
\[ 2.22 \text{ MeV} \]

Tritium \( \text{^3H} \)
\[ 6.92 \text{ MeV} \]

Helium-3 \( \text{^3He} \)
\[ 7.42 \text{ MeV} \]

Helium-4 \( \text{^4He} \)
\[ 28.3 \text{ MeV} \]

Carbon-12 \( \text{^{12}C} \)
\[ 92.2 \text{ MeV} \]

\( A = 2+4N \)

\[ Z = 1, 2 \]

Simple energy considerations would suggest that these nuclei will be produced when \( T \approx 1-30 \text{ MeV} \); however, this is not the case because the baryon to photon ratio is so small \( n_B/n_\gamma \approx 10^{-10} \). That there are a lot of photons around:

\[ n_\gamma = 2.68 \times 10^{-8} \text{ s}^{-2} \text{cm}^{-4} \]

\( n_B/n_\gamma \approx 10^{-5} \)

In E\( \gamma \) (assumed reaction are fast enough), elements with \( A \) will have abundance:

\[ n_A = n_\gamma \left( \frac{m_{\gamma} T}{2\pi} \right)^{3/2} \exp \left[ -\frac{m_{\gamma} + m_A}{T} \right] \]

The \( n_p \) abundance:

\[ n_p \sim 2 \left( \frac{m_{\gamma} T}{2\pi} \right)^{3/2} \exp \left[ -\frac{m_{\gamma} + m_p}{T} \right] \]

\[ n_n \sim 2 \left( \frac{m_{\gamma} T}{2\pi} \right)^{3/2} \exp \left[ -\frac{m_{\gamma} + m_n}{T} \right] \]

In chemical equilibrium, the chemical potentials are related as:

\[ \mu_A = \mu_p + (A-Z)\mu_n \]

\[ e^{\mu_A/T} = e^{[\mu_p + (A-Z)\mu_n]/T} = n_p \frac{Z}{n} n_{A-Z} \left( \frac{2\pi}{m_{\gamma} T} \right)^{3/2} 2^A e^{\frac{Z m_p + (A-Z) m_n}{T}} \]

\[ n_A = n_p A^{3/2} \left( \frac{2\pi}{m_{\gamma} T} \right)^{3/2} \frac{Z}{A} n_{A-Z} \exp \left[ \frac{Z m_p + (A-Z) m_n}{T} \right] \]

Since particle number densities decrease as \( A^{-3} \) (for const. number per volume), it is useful to normalize by the total baryon density
look at mass fraction:

\[ X_A = \frac{A \eta_A}{n_B} \]

where \( n_B \) is baryon density \( \sum_i \eta_i = 1 \)

\[ \eta_p = \eta_B (X_A / A) = \eta \eta (X_A / A) \]
\[ \eta_p = \eta_B \chi_p = \eta \eta \chi_p \]
\[ \eta_n = \eta_B \chi_n = \eta \eta \chi_n \]

\[ \eta \approx 2.2 (3) \times 10^{-7} \] \( T_0^3 \)

\[ \eta \approx 2.68 \times 10^{-7} \] \( B_0 h^2 \) i.e. present cosmological value

\[ X_A = \left[ 9 \left( 1 + \frac{3}{2} (34.5)^{1/2} \right) - 1 \right]^{-1} \eta^{-1} \]
\[ \eta \approx 2.68 \times 10^{-7} \]

\[ X_A = X_p \approx X_n \approx \exp \left( \frac{B_0}{T_0} \right) \]

\[ T_{1/2} \approx \frac{B_0}{(A+1)} \ln (\eta^{-1}) + \frac{3}{2} \ln (T_0) \]

So, \( \eta < 1 \) need \( T \gtrsim B_0 \) to have a reasonable large abundance.

An estimate we can get being \( \chi_p \approx \chi_n \) => \( X_A \approx 1 \) when

\[ T_{1/2} \approx \frac{B_0}{(A+1)} \ln (\eta^{-1}) + \frac{3}{2} \ln (T_0) \]

\[ \begin{align*}
  ^2 \text{H} & : 0.07 \text{ MeV} \\
  ^3 \text{He} & : 0.11 \text{ MeV} \\
  ^4 \text{He} & : 0.28 \text{ MeV} \\
  ^{12} \text{C} & : 0.25 \text{ MeV}
\end{align*} \]

So, significant production only happens at much lower energies.

Thus far, we assumed equilibrium.

There are 2 crucial general type of reactions that determine the abundance of elements:

i) \( n / p \) determined by weak interaction rate

ii) nuclear reaction rate
we have to check when these reactions are in equilibrium.

i) p-n reaction by:

\[
\begin{align*}
\begin{cases}
\nu \rightarrow p + e^- + \bar{\nu} \\

\nu + n \rightarrow p + e^-
\end{cases}
\end{align*}
\]

\[e^+ + n \rightarrow p + \bar{\nu}\]

When things are in EQ (T >> H), \( \mu_n + \mu_\nu = \mu_p + \mu_e \) and then:

\[
\left( \frac{n}{p} \right)_{eq} \approx \left( \frac{n}{p} \right)_{eq} = \left( \frac{\rho_e}{\lambda_p} \right)_{eq} = \exp \left[ -\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T} \right] \approx \exp \left[ -\frac{Q}{T} \right]
\]

recall \( \alpha \approx 1.3 \text{MeV} \), so as temperature drops below this, protons are favoured over neutrons.

To see how long this ratio is maintained as universe cools down, need to calculate \( T \) for the interactions above (using weak interactions) - The result is that \( (T_{1/2} \text{MeV}) \)

\[
\frac{T}{H} \sim \left( \frac{T}{0.8 \text{MeV}} \right)^3
\]

so at \( T > 0.8 \text{MeV} \) \( \frac{n}{p} = (\frac{\rho}{p})_{eq} \), at \( T < 1 \text{MeV} \) \( \frac{n}{p} \approx (\frac{\rho}{p})_{eq} \).

When \( T \) drops below 0.8, the neutron to proton ratio "freezes out" (decays), and stays constant - however, since neutron is unstable, it actually does not stay exactly constant but decays with half life \( \approx 10 \) minutes. So, when \( BBN \) starts, there will be a lot less neutrons than from \( (n/p)_{eq} \).
ii) The nuclear reaction rates can be shown to be in EQ. \( T > 1 \text{ MeV} \) at temperatures of 1 MeV, so at this \( T \), \( n, p, \delta^5, \nu^5, \psi^5 \) are in EQ at the same temperature. But as mentioned before at \( T \approx 1 \text{ MeV} \) the \( X_p \) are very small.

As \( T \) drops below \( T \approx 1 \text{ MeV} \), many interesting things happen.

1) \( \nu \) decouples @ \( T \approx 1 \text{ MeV} \)
2) \( \Delta \), etc. annihilate and transition entropy to \( \delta^5 \), at \( T \approx 0.5 \text{ MeV} \)
3) \( n \approx 0.8 \text{ MeV} \), \( X_p \) freeze-out with a value

\[
\left( \frac{N}{P} \right)_{\text{freeze-out}} = \exp \left( -\frac{q}{T_F} \right) = \exp \left( -\frac{1.3}{0.8} \right) \approx \frac{1}{6}
\]

at this time, nuclear reactions are still in EQ, but abundances are small,

\[
X_n \approx \frac{1}{7} \\
X_P \approx \frac{6}{7}
\]

Deuterium \( X_2 \approx 10^{-12} \)

Tritium \( X_3 \approx 10^{-23} \)

\( ^{4}He \) \( \delta_4 \approx 10^{-23} \)

- When \( T \) drops to \( T \approx 0.3 \text{--} 0.1 \text{ MeV} \) \( (t = 1 \text{--} 3 \text{ minutes}) \) \( \text{NUC} \) takes place

Some neutrinos decay by decay

\[
\exp \left( -\frac{3 \text{min}}{10 \text{min}} \right) \approx 0.8 \\
\left( \frac{N}{P} \right)_{\text{NUC}} \approx \frac{1}{6} \times 0.8 \approx \frac{1}{7}
\]
[recall the EQ value would be \( (\frac{n}{p})_{eq} = \exp(-\frac{1.3}{0.3}) \approx \frac{1}{76} \)]

at \( T \approx 0.3 \text{ MeV} \) X-ray bremsstrahlung of order unity; however, the fastest way to create \(^4\text{He}\) is through deuteronium:

\[
\begin{align*}
2\text{H} + 2\text{H} & \leftrightarrow \text{He} + p \\
2\text{H} + 2\text{H} & \leftrightarrow \text{He} + n \\
3\text{H} + \text{He} & \leftrightarrow \text{He} + \text{He} + n \\
3\text{He} + \text{He} & \leftrightarrow \text{He} + \text{He} + p
\end{align*}
\]

Since deuteronium is formed by

\[ n + p \leftrightarrow \text{He} + \gamma \]

it has actually to wait until \( T \approx 0.1 \text{ MeV} \) when photodissociation of \( \text{He} \) by \( \gamma \)'s is low enough (recall \( B_{\gamma\gamma} \approx 2.2 \text{ MeV} \)); again, there are lots of photons around.

\[
\text{Similarly, EQ value for } ^3\text{He} \text{ are very small at } T \approx 0.3 \text{ MeV}
\]

When \( T \approx 0.1 \text{ MeV} \), \( ^2\text{H}, ^3\text{He} \) lead to \(^4\text{He}\), and essentially all reactions end up in the most bound state, \(^4\text{He}\) (this is only an approx., but close). Then we can estimate the mean fraction of \(^4\text{He}\) by:

\[
X_{^4\text{He}} = \frac{4}{n_n} = \frac{4}{n_n/2} = \frac{2(n/p)_{\text{Nuc}}}{1 + (n/p)_{\text{Nuc}}} \approx \frac{1}{4}
\]

As reactions proceed, \(^2\text{H}, ^3\text{He}\) get depleted, and since \( T \approx 0.1 \text{ MeV} \), and \( n/n_p \) rates go down and freeze out,
leaving residual fraction of deuterium and 3He. Since $N_\alpha \propto X_\alpha \eta^{3/2}$, these abundances depend on $\eta$ sensitively. On the other hand, $^4\text{He}$ depends mostly on $(O/P)_{\text{nuc.}} - (O/P)_{\text{Thallium}}$.

- What happens with heavier elements? They are suppressed:
  
  i) By the time deuterium is available to form $^3\text{He}$, the Amontons barrier becomes large compared to energy ($\text{depends on } \frac{\sqrt[1/3]{A+2}}{4\pi^2} \frac{T^{-1/3}}{(\text{TeV})}$; $A = \frac{A_1 A_2}{A_3}$)

  ii) There are no tightly bound elements with $A = 5, 8$

  iii) The density is low enough that the triple $\alpha$ reaction $^3\text{He} \rightarrow ^{12}\text{C}$ is strongly suppressed (this works in stars though)

  iv) Some traces of $^7\text{Li}$ and $^7\text{Be}$ are produced

$$\frac{^7\text{Li}}{\text{H}} \sim 10^{-9} - 10^{-10}, \quad \frac{^7\text{Be}}{\text{H}} \sim 10^{-11}$$

- The residual fraction of deuterium and $^3\text{He}$ is $D$, $^3\text{He}/\text{H} \sim 10^{-4}$

[Show transparency with results of SN36L]

$$\eta \sim (3-10^2) \times 10^{-10} \Rightarrow \sigma_{\text{Bb}}^2 \approx 0.02 \pm 0.01$$