Supernovae Results

We now turn to a discussion about the recent evolution of the universe, and
the evidence that has grown in the last few years that we are
currently undergoing cosmic acceleration, somewhat similar to an
inflationary epoch.

The classic test for measuring the acceleration of the universe is

to measure the luminosity distance \( d_L \) as a function of redshift, which
is sensitive to \( \Lambda \) and its derivatives. The luminosity distance
\( d_L \) is defined by

\[
d_L = \frac{L}{4\pi F}
\]

where \( L \) and \( F \) are the intrinsic luminosity and observed flux of
a given object within a specified passband. Sometimes things
are expressed in magnitudes, which are logarithmic measures of the
flux (apparent magnitude \( m \)) and the luminosity (absolute magnitude \( M \)),

\[
m = -2.5 \log F + \text{const.}
\]

where \( \text{const.} \) is fixed by units of
flux and band.

What we care about is \( m - M \), called distance modulus, which does not
depend on \( \text{const.} \); \( M \) is defined as \( m @ 10 \text{ pc} : \)

\[
m = -2.5 \log \left[ \frac{L \ (\text{L}_\odot \ \text{yr}^{-1})^2}{M} \right] = -2.5 \log L - 5 \log \frac{\text{L}_\odot \ \text{yr}^{-1}}{M}
\]

\[
\Rightarrow m - M = 5 \log d_L (\text{Mpc}) + 25 \quad \text{(distance modulus)}
\]

In order to determine \( d_L (z) \) we need to know \( L \) (or \( M \)) and
\( F \) (or \( m \)). Flux (or apparent magnitude) is easy, but
the tricky part is in obtaining the intrinsic luminosity
(or abs. magnitude) of a source. A source with constant \( L \)
is said to be a standard candle. If we can find
such class of objects in the universe as a function of $z$, then we can determine $d_L(c)$. 

Recently, significant progress has been made using Type Ia supernovae as "standardizable candles". Supernovae are rare (a few every galaxy) but they are very bright ($M_{-19.5}$), typically comparable to host galaxy, and thus they can be seen to high redshifts ($z<1$) which makes cosmological evolution. 

**Why type Ia supernovae?** To understand a bit about this, let's take a look at the classification of supernovae:

$$
\text{Type Ia} \rightarrow \text{Type Ib} \rightarrow \text{Type Ic} \\
\text{Occur in all types of galaxies (most frequently in spiral galaxies)}
$$

- **Type Ia**
  - Presence of hydrogen in their spectra
  - Strong Si II
  - Occurs in all types of galaxies
  - Pre: Carbon-oxygen white dwarfs (WD) that accrete matter from a companion star and undergo thermonuclear runaway. Probably the WD reach Chandrasekhar limit before exploding. The usefulness of type Ia rests on this, $M_{1.4} = 1.4 M_\odot$ being a nearly universal quantity.

- **Type Ib**
  - Presence of hydrogen in their spectra
  - Prominent He I

- **Type Ic**
  - No Si II/He I
  - Do not occur in elliptical galaxies
  - Typically in or near spiral arms and HII regions

Pre: massive stars ($M > 8-10 M_\odot$) that suffer core collapse (generally iron) and then rebound leaving a neutron star or a black hole. 

In Type Ib-c progenitors are thought to be star-paired or their hydrogen ($Ib$) and helium ($Ic$) prior to exploding, either by mass transfer to companion stars or through winds.
Although ideally one expects Type Ia to be standard candles, resulting from $M_{1.4}c^2$ energy worth / explosion, in practice there is $\sim 40\%$ scatter in the peak brightness of nearby supernovae, which may be traced perhaps to difference in the composition of the WD atmospheres. However, the observed difference in peak luminosities are correlated with the shape of the light curves: dimmer supernovae decline faster, thus measuring the decline light curve one can empirically correct for this, decreasing the scatter from $40\%$ to $\sim 15\%$. It appears that physically the main reason for this can be traced to the amount of $^{56}Ni$, produced in the supernova explosion; more $^{56}Ni$ implies higher peak luminosity and thus higher temperature and opacity which leads to a slower decline of the light curve.

Let's go back to $d_L(z)$. We will work in a spatially flat universe for simplicity ($\Omega_{m} = 1$), then it follows that

$$d_L = c \int_{0}^{z} \frac{dz'}{H(z')}$$

You can derive this from the expression of $d_L$ we obtained in Section 4. This is left as an exercise. Let's expand this about today ($t_0$), then

$$\frac{a(t)}{a_0} \approx 1 + H_0 (t-t_0) + \frac{1}{2} H_0^2 (t-t_0)^2 + \frac{1}{3} \frac{\Omega_m H_0^2 (t-t_0)^3}{a_0} + \cdots$$

with

$$\begin{align*}
H(t) &= \frac{\dot{a}}{a} \\
q(t) &= -\frac{\ddot{a}}{a} - \frac{1}{H^2} \\
j(t) &= \frac{\dddot{a}}{a} + \frac{1}{H^3}
\end{align*}$$

Hubble constant

acceleration parameter

"jerk" parameter

Then:

$$d_L(z) \approx \frac{c^2}{H_0} \left[ 1 + \frac{1}{2} (1-q_0) z - \frac{1}{6} \left[ 1 - q_0 - 3 q_0^2 + 5 q_0 \right] z^2 + O(z^3) \right]$$

Thus, acceleration manifests as a quadratic correction in $d_L(z)$, whereas change of acceleration leads to a cubic term.
A week ago the first reports of reliable detection of change in acceleration appeared in astro-ph. Another way of parameterizing this change in acceleration is to look at \( \frac{d\Omega}{dz} \). From this one can define a redshift of transition (from deceleration in the past to acceleration now) as \( q(z_t) = 0 \) then

\[
\frac{z_t}{z_0} \approx \frac{d\Omega}{dz} \approx 0.46 \pm 0.13
\]

to the transition happened fairly recently, meaning that \( w \) cannot be too far from \(-1\), otherwise (for fixed \( \Omega_m \) and \( \Omega_{\Lambda} \)) time evolution of dark energy means it was important in the past (I'm assuming \( w \neq -1 \) here, for \( w = -1 \) it's important that \( z_t > 0.33 \)).

[Show results from SN Ia]