Homework Set #1  (Due 2/28 in Class)

1. Show that the luminosity distance can be written in an arbitrary FRW universe as

\[
d_L(z) = \frac{(1 + z)}{H_0 \sqrt{\Omega_k^0}} \sinh \left( H_0 \sqrt{\Omega_k^0} \int_0^z \frac{dz'}{H(z)} \right),
\]

where \( \Omega_k^0 = -k/(a_0 H_0)^2 \) is today’s contribution of spatial curvature to the critical density.

2. Show that the equation of state parameter of dark energy \( w = p_{DE}/\rho_{DE} \) can be written as

\[
w(z) = \frac{(1 + z)(2/3)HH' - H^2}{H^2 - \Omega_m^0 (1 + z)^3 H_0^2},
\]

assuming there is negligible radiation and that the universe is spatially flat. \( H' \) denotes \( dH(z)/dz \).

3. Show that vacuum quantum fluctuations obey Gaussian statistics.

\textit{Hint}: Expand each mode \( \phi_k = w_k a_k + w_k^* a_k^\dagger \) in terms of creation and annihilation operators \( a_k^\dagger \) and \( a_k \), obeying the usual commutation relations and calculate the moment generating function for such a mode. For operators \( A \) and \( B \) that commute with their commutator \([A, B] \), the following may come handy: \( e^{A+B} = e^A e^B e^{-\frac{1}{2} [A, B]} \).