Homework Set #2  (Due 4/9 in Class)

1. The “horizon problem”.
   a) Show that in the standard Big Bang model the horizon distance today is to a very good approximation \( d_H \approx 2H^{-1} \), even though the universe changed from radiation to matter dominated sometime in the past. Assume that \( \Omega_{m}^{(0)} = 0.3 \) and that the CMB temperature today is 2.7° K.

   b) Consider now the evolution of \( d_H \) and \( H^{-1} \) when there is a period of inflation. Assume the universe is radiation dominated before and after the period of inflation, which lasts 60 e-folds. Show that the horizon and the Hubble radius are very different today. Explain why the behavior of \( d_H \) solves the “horizon problem”. Sketch a graph of scales, \( H^{-1} \), and \( d_H \) as a function of scale factor.

   c) Show that once scales are in causal contact (as measured by \( d_H \)) they stay forever in causal contact.

   d) The explanation in b) is not the usual explanation found in discussions about inflation (e.g. in books), which actually involves \( H^{-1} \) rather than \( d_H \). Why is this so, and why does the “standard” explanation make sense?

2. Show using stress-energy conservation that in the spatially-flat gauge, the equations of motion for the inflaton perturbations correspond to a free field

   \[
   \ddot{\delta \phi} + 2H \dot{\delta \phi} + k^2 \delta \phi = 0, \tag{1}
   \]

   to leading order in the slow-roll parameters.

3. Assuming the slow-roll approximation, find the most general form of the inflaton potential so that scalar perturbations are exactly scale-invariant.

4. In local primordial non-Gaussianity the primordial curvature perturbation \( \zeta \) can be written in terms of a Gaussian field \( \zeta_G \) by the following local map in real space,

\[
\zeta(x) = \zeta_G(x) + \frac{3}{5} f_{NL}^{\text{loc}} \left[ \zeta_G(x)^2 - \langle \zeta_G(x)^2 \rangle \right], \tag{2}
\]

Show that this leads to the local form for the bispectrum in Fourier space (to leading order in \( f_{NL}^{\text{loc}} \)).
\[
B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}^{\text{loc}} \left[ P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right]
\] (3)