1. Stellar collisions

a. From basic trigonometry:
\[ R_\odot = D_\odot \tan \theta \approx D_\odot \theta \]  
(1)
(where \( \theta \) is in radians). So
\[ R_\odot \approx (1 \text{ AU})(0.5 \text{ deg}) \frac{\pi}{180 \text{ deg}} \approx 10^{11} \text{ cm} \]  
(2)

b. From the top you can approximate both galaxies as uniform filled circles of stars. Thus, their areas are \( A \sim \pi r^2 \), with \( r \sim 10 \text{ kpc} \). With \( 10^{10} \) stars each, this yields, 30 stars per parsec\(^2\). From the side, approximate the galaxies rectangles that are 300 pc by 20 kpc. This yields a surface density of 1600 stars per parsec.

c. The surface of each star covers an area of about \( A \sim \pi R_\odot^2 \sim 3 \times 10^{22} \text{ cm}^2 \). In parsec units this is:
\[ A \sim \frac{(3 \times 10^{22} \text{ cm}^2)}{(3 \times 10^{18} \text{ cm/pc})^2} \sim 3 \times 10^{-15} \text{ pc}^2 \]  
(3)
So from above, the fraction of the galaxy covered by surfaces of stars is \( 10^{-13} \), and from the side it is about \( 2 \times 10^{-11} \).

d. The orientation that maximizes the number of collisions is that which maximizes the fractional area covered by stars, which is from the side. The chances of a collision per unit parsec is the fraction of area covered by stars, times the density of stars, which is \( 10^{-8} \) in the case that they approach each other from the side. The total area covered by the collision is \( 6 \times 10^6 \text{ pc}^2 \). So the number of expected collisions is \( \sim 10^{-1} \). So if the Milky Way and Andromeda collide roughly zero stellar collisions are expected.

e. There are number of effects which might enhance the rate of collisions. For one, galaxies aren’t uniform, they have big concentrations near their centers. But in addition, stars affect each other by gravity as well, and gravitational focusing can enhance their effective cross sections.

2. Motion around the Galactic Center

a. The period of a circular orbit is of course:
\[ P = \frac{2\pi r}{v} \]  
(4)
and the velocity of a gravitationally bound star in such an orbit is:
\[ v = \sqrt{\frac{GM}{r}} \]  
(5)
So:

\[ P = 2\pi \sqrt{\frac{r^3}{GM}} \]  

(6)

For \( r \sim 0.005 \text{ pc} \) and \( M \sim 10^6 \), this evaluates to \( P \sim 20 \text{ yr} \). So the time scale for this orbits is comparable to human time scales, making it possible to measure. Note that \( P \propto \rho^{-1/2} \) in terms of the density interior to the orbit \( \rho \sim M/r^3 \). This scaling is very general and crops up in many applications in gravitational dynamics.

b. If we don’t know the location of the black hole, an elliptical orbit can either be circular, but seen from an angle, so appearing flattened, or intrinsically elliptical. For a circular orbit, the velocity is constant, so that at each extreme of the ellipse we will measure equal absolute values of the velocity. On the other hand, for an orbit that is intrinsically elliptical, the velocity extremes will differ: the velocity will be greatest at closest approach. Thus, you can see how one can use velocity measurements to learn about the real orientation of the orbit.

c. If \( \alpha \) is the radius of the orbit, then the semimajor axis \( a = \alpha \) and the semiminor axis \( b = \alpha \sin \theta \), where \( \theta \) is the angle between the line of sight and the plane of the orbit. So \( b/a = 0.5 \) when \( \sin \theta = 0.5 \), which is when \( \theta = 30 \text{ deg} \).

d. On the sky, if the semimajor axis is \( \alpha \), then \( \alpha \approx r/D \), where \( r \) is the radius of the orbit in centimeters and \( D \) is the distance from us to the black hole. From above, we know that \( P = 2\pi r/v \), so \( D = vP/(2\pi \alpha) \). Thus, if we know \( v \) and \( P \) we can easily calculate \( D \).

e. Of course, one just reverses the logic in part (a), and with the measured radius and the orbit and velocity calculate the black hole mass \( M = rv^2/G \).

If you looked up the paper, you’d notice that Eisenhauer et al. (2003) don’t solve their problem this way. In their more complex situation, they have a bunch of data points: times, positions, and velocities. They construct a mathematical model which requires as input the mass of the black hole, the distance to the black hole, the ellipticity of the orbit, and a number of other parameters. By choosing the appropriate inputs to the model, they can explain their data points. You would also notice that, because spectra of these stars are difficult to obtain, they only have the velocity at three moments during the orbit! Still, by providing extra constraints on the true orientation and shape of the orbit, the velocities are very important to the determination.

REFERENCES


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