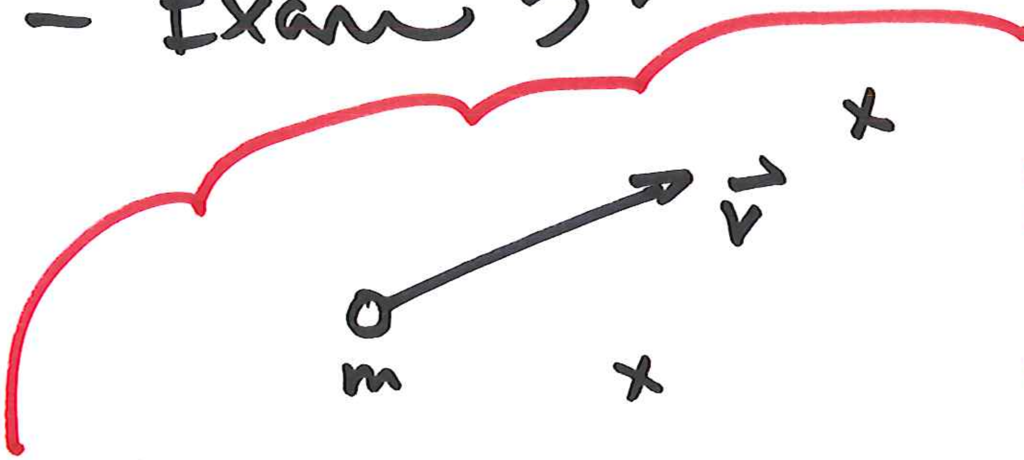
NYU Physics I 2018-11-20

- No QS.
- a, e.
- Exam 5.

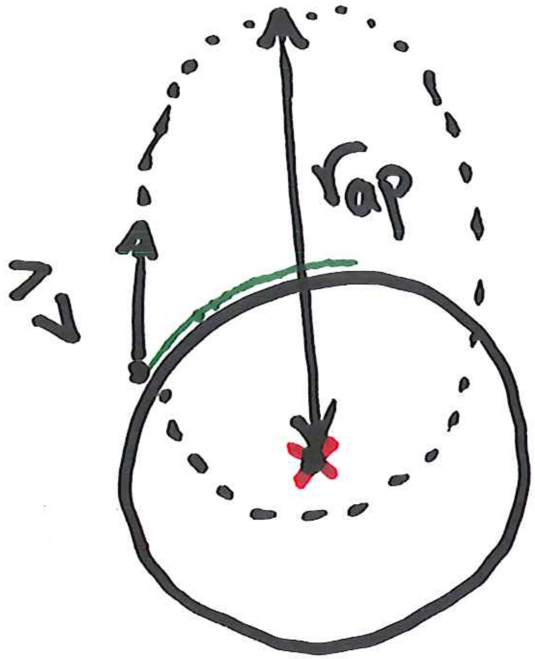
- ellipses – a, e.
- Kepler's laws.
- Newtonian gravity.



spin $\vec{L} = I\omega$

orbit $\vec{L} = m\vec{r}\times\vec{v}$

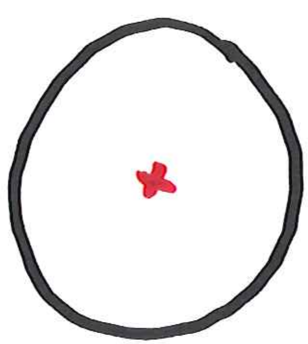
$= \vec{r}\times\vec{p}$



$\vec{L}$ is invariant!

- Force is radial

so $\frac{dL}{dt} = \tau = 0$

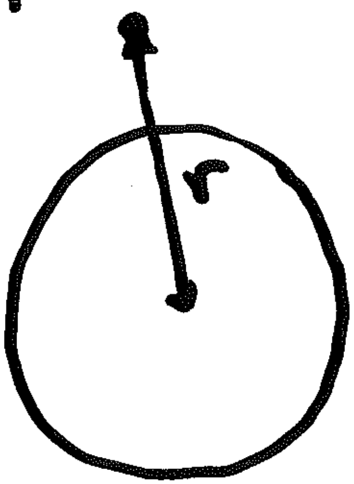


$E = \frac{1}{2}mv^2 + mgh$

$= \underbrace{\frac{1}{2}mv_x^2}_{\text{invariant}} + \underbrace{\frac{1}{2}mv_y^2}_{\text{zero @ apex.}} + mgh$ (A)

h_{max} = "soln of
when $v_y = 0$."

potential energy $U(r) \neq mgr$



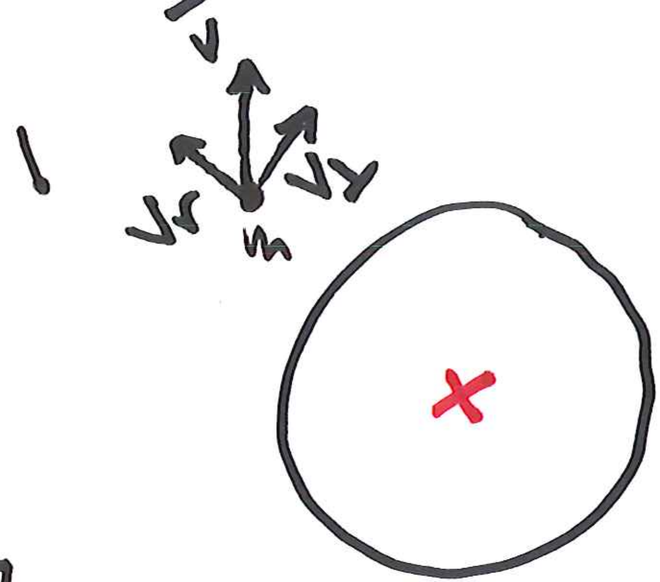
$$U(r) = ☹ \, \frac{GMm}{r} \qquad ☹ \equiv -1$$

$$F_r(r) = -\frac{GMm}{r^2} = -\frac{d}{dr}\left[-\frac{GMm}{r}\right]$$

$$F_r = \cancel{-\frac{d}{dr}} U \quad -\frac{d}{dr} U$$

total energy $\boxed{E = \frac{1}{2}mv^2 - \frac{GMm}{r}}$

$$E = \frac{1}{2} m v^2 - \frac{GMm}{r}$$



$$E = \frac{1}{2} m v_r^2 + \frac{1}{2} m v_\perp^2 - \frac{GMm}{r}$$

$$\boxed{E = \frac{1}{2} m v_r^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}}$$

$$U = \frac{A}{r^2} - \frac{B}{r}$$

$$L = m r v_\perp$$

$$|\vec{r} \times \vec{v}| = |\vec{r}| v_\perp = |\vec{v}| r_\perp$$

$$v_\perp = \frac{L}{mr}$$

$$\frac{1}{2} m v_\perp^2 = \frac{L^2}{2mr^2}$$

At apocenter, pericenter, $\boxed{v_r = 0}$

$$E = 0 + \frac{L^2}{2mr_{ap}^2} - \frac{GMm}{r_{ap}}$$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$0 = \underbrace{2mr^2E}_{a} - \underbrace{L^2}_{c} + \underbrace{GM2m^2}_{b}r$$

$$r = \frac{-GM2m^2 \pm \sqrt{G^2M^24m^4 + 8L^2mE}}{4mE}$$

$$r = \underbrace{\frac{2GMm}{4E}}_{\text{semi-major axis??}} \pm \sqrt{\frac{G^2M^2m^2}{4E^2} + \frac{1}{2}\frac{L^2}{mE}}$$

$$\left(\frac{1}{4}\right)^{1/2} = \frac{1}{2} \qquad \left(\frac{1}{4}\right)^{2} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{3} = \frac{1}{64.} \qquad \left(\frac{1}{4}\right)^{\frac{1}{3}} = \frac{1}{1.6}$$