

# NYU Physics I—Problem Set 14

Due Thursday 2018 December 13 at the beginning of lecture.

**Problem 1:** (a) The kinetic energy of a particle is defined to be the difference between the total energy  $E$  given by

$$E^2 = p^2 c^2 + m^2 c^4 \quad (1)$$

(where  $p$  is the relativistically correct momentum formula) and the rest-mass energy  $E_0$  given by

$$E_0 = m c^2 \quad . \quad (2)$$

Given these formulae, make a table showing the kinetic energy of a 2 kg mass moving at the following set of speeds:

$1.0 \times 10^{-2} \text{ m s}^{-1}$  (you might have to use an approximation for this one)

$1.0 \times 10^0 \text{ m s}^{-1}$  (here too maybe)

$1.0 \times 10^2 \text{ m s}^{-1}$

$1.0 \times 10^4 \text{ m s}^{-1}$

$1.0 \times 10^6 \text{ m s}^{-1}$

$1.0 \times 10^8 \text{ m s}^{-1}$

$2.0 \times 10^8 \text{ m s}^{-1}$

$2.9 \times 10^8 \text{ m s}^{-1}$

$0.99 c$

$0.9999 c$

Give your answers in scientific notation, to seven decimal places, in Joules. Feel free to use a computer program or spreadsheet to calculate and format this table.

(b) Add to your table a column giving the non-relativistic formula for kinetic energy we learned weeks ago. Do you see any issues with your calculations, or are you okay with them? (For context, the first four involve the differences of extremely large, very similar numbers!) If it needs fixing, can you figure out how to fix it?

(c) Add to your table a column giving the ratio of the relativistic kinetic energy to the output of the non-relativistic formula. Again, to 7 decimal places. Feel free to tweak everything until it all looks right.

**Problem 2:** From the notes at <http://cosmo.nyu.edu/hogg/sr/>, Problem 4–8.

**Problem 3:** From the notes at <http://cosmo.nyu.edu/hogg/sr/>, Problem 4–11.

**Problem 4:** From the notes at <http://cosmo.nyu.edu/hogg/sr/>, Problem 6–10.

**Extra Problem (will not be graded for credit):** (a) Forgetting about Special Relativity, and assuming just Newtonian mechanics, compute how long you would have to accelerate at acceleration  $g = 10 \text{ m s}^{-2}$  in order to reach the speed of light.

(b) A relativistically correct constant-acceleration trajectory on a space-time diagram is a hyperbola, where both asymptotes are 45-degree lines (null trajectories). Find a formula (position  $x$  as a function of time  $t$ ) for this hyperbola, constrained to have acceleration  $g$  at small times.

(c) Show that this trajectory is unchanged under the Lorentz transformation. That is, show that if you boost in the  $x$  direction, the trajectory translates onto itself (except, possibly, for a small shift in the  $x$  or  $t$  direction).