Kinematic snapshot, dynamical inference

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we are developing \textit{orbital inference} methods for integrable, phase-mixed dynamical systems

- snapshot in time of $(\mathbf{x}, \mathbf{v})$; what is the potential?
- example: Solar System
- similar to virial estimators but much more precise

when phases aren’t mixed, we are using anisotropies that develop in phase space to perform dynamical inference

- cold stellar streams
- example: Grillmair stream
- action space looks different from angle space

when phases aren’t mixed \textit{and} potentials aren’t integrable, phase-space structure is still highly informative

- \textit{perturbed} cold stellar streams
- comprehensive modeling from $t = 0$
dark matter and the Galaxy

- the Milky Way is the best place to study dark matter at small scales and in the nonlinear regime
- if dark matter annihilation is tentatively detected, can we confirm the detections through dynamical tests?
- we expect extremely rich structure in the Galaxy’s dark sector; if there is no annihilation signal, dynamics is our only tool
The *Gaia* Mission

- precise astrometric observatory
- tens of microarcsecond \((10^{-10} \, \text{rad})\)
- 5-\(d\) measurements on \(10^9\) stars
- 6-\(d\) on \(10^8\)
- goals include dynamical state and assembly history of the Milky Way
kinematic snapshot

- *Hipparcos* and *Gaia*: stellar positions and velocities measured in few-year missions
- Orbital times of order $10^8 \text{ yr}$
- Don’t see any orbital curvature or accelerations at all
  - Dynamical inference must be indirect
  - Kepler inferred ellipses from measurements spanning a full dynamical time
  - Newton inferred $(1/r^2)$ force law from *shapes* of orbits; no chance of that here
reminder about integrability

- integrable potentials have symmetries that lead to conserved quantities
- in three dimensions, three conserved quantities
- canonical transformation to action–angle coordinates
  - actions are invariant
  - angles increase linearly with time
  - each orbit (set of three actions) fills a three-torus
  - (except resonant situations, which make one- and two-tori)
mixed angles

- put many test particles on orbits in an integrable potential
- wait many dynamical times
- all original structure in the angles will eventually disappear
- distribution in \((x, v)\) space will be a function of actions only
Schwarzschild and Oort programs

- measure every stellar position and velocity you can
- loop over integrable potentials, computing actions for all stars
- does a set of angle-mixed populations generate the potential?
- does it also generate the observed actions?
- not very practical:
  - small samples
  - selection effects and measurement noise
  - dark matter
  - non-integrability and unmixed angles
  - computation
baby problem: Kepler problem

- can you determine the mass of the Sun with a snapshot of nine (or seven) positions and velocities?
- can you determine the \((1/r^2)\) force law?
- of course you can!
  - loop over all possible masses and (integrable) force laws
  - compute actions and angles from positions and velocities
  - prefer masses and force laws for which the angles “look mixed”
- this problem is called “orbital roulette” (Beloborodov & Levin)
comparison: virial theorem

- The mean kinetic energy (time-averaged over an orbit) equals a dimensionless factor $Q$ times the mean potential energy for any given potential

\[
\langle K \rangle_t = -\frac{1}{2} \langle U \rangle_t
\]

- If you assume mixed angles, then the mean kinetic energy (averaged over test particles) equals $Q$ times the mean potential energy

\[
\langle K \rangle_{\text{objects}} = -\frac{1}{2} \langle U \rangle_{\text{objects}}
\]
if you know the force law, you can determine the mass of the Sun by just setting the total kinetic energy to be $Q = -(1/2)$ times the total potential energy

this method is imprecise (probably because the $N$ test particles are used *additively*)

works trivially with *missing data*. 
frequentist orbital roulette

- what potential is consistent with the test particles having randomly selected angles?
- if you set the mass too small, everything is really close to perihelion
- if you set the mass too large, everything is really close to aphelion
- use some distribution sampling test (such as K-S or A-D) to test consistency of any mass (or potential) with the mixed-angles assumption.
- problems include
  - essentially never have all six dimensions of \((x, v)\)
  - always have significant measurement uncertainties
  - usually have selection effects
what are we doing?

- putting orbital roulette into an inference framework (read: Bayesian) that permits measurement uncertainties, missing data, and selection effects
- applying to extant data on the Galactic Center
  - heterogenous data at different radii
  - significant errors and missing data
  - inconsistent masses in literature
- generalizing from the Kepler problem to galaxy potentials
- also the Oort problem (vertical structure in the disk)
- with Bovy (NYU), Lang (Toronto), Murray (Toronto), Tremaine (IAS)
example: If Newton had only a snapshot...

- Bovy, Murray, Hogg, arXiv:0903.5308
- take 8-planet ephemeris snapshot at 2009 April 1.0
- \( \mathbf{a} = -A \left[ \frac{r}{r_0} \right]^{-\alpha} \)
- \( f_\theta(\mathbf{x}, \mathbf{v}) \propto |J| f_\theta(\ln \epsilon, e) \)
- Bayes theorem
but angles are *never* mixed: theory

- the Milky Way has been around for $10^2$ dynamical times
- stars are formed in groups with velocity widths of a few km s$^{-1}$
- Milky Way orbits are 200 km s$^{-1}$
- angle-mixing will be complete only after much more than $10^2$ dynamical times
- *and* new stars are forming all the time
Recap

- *First* we assumed integrable potential and mixed angles
- *Now* we will assume integrable potential but *unmixed angles*
angles are *never* mixed: observation

- substructure and satellites
- tidal streams
- velocity structure in the disk
Fig. 1.— Smoothed, summed weight image of the SDSS field after subtraction of a low-order polynomial surface fit. Darker areas indicate higher surface densities. The weight image has been smoothed with a Gaussian kernel with $\sigma = 0.2^\circ$. The white areas are either missing data, or clusters, or bright stars which have been masked out prior to analysis.

(Grillmair & Dionatos 2006)
cold streams: origin

- globular cluster or compact satellite orbiting Galaxy
- at the tidal radius, stars peel off
- each star is on an orbit very close to the satellite orbit
cold streams: ideal properties

- cold: internal velocity dispersion of the satellite was small
- mass was also small so the tidal radius was small
- in the limit, the stars on the stream are on the *same* orbit
  - that is, same actions
  - but...
- the stars are spread out in angle *only*
- (recall the approximation: integrable potential)
cold streams: configuration

- working in the *cold limit*, the stream traces out a segment of an orbit
- the *plane* of the stream in configuration space, locally, is also a plane containing the acceleration vector
- just the *shape* of the stream puts limits on *oblateness* of the Galaxy
- we have done this for the Grillmair stream
- with Koposov (MPIA), Rix (MPIA)
Fig. 1.— The number density of SDSS stars with $0.15 < g - r < 0.41$ and $18.1 < r < 19.85$ in the rotated coordinate system with the pole at $(\alpha, \delta) = (34.5987, 29.7331)$. $\phi_1, \phi_2$ are the longitude and latitude in a new coordinate system. The map was convolved with a circular Gaussian with $\sigma = 0.2$ deg. The stream is barely visible on the image and is going horizontally near $\phi_2 = 0$. 
FIG. 2.—Profile in stars with $0.15 < g-r < 0.41$ $18.1 < r < 19.85$ across the $\phi_2 = 0$ axis. The dotted line shows the profile of stars of all stars with $-70 < \phi_1 < 10$. The solid line shows the weighted profile of stars $-70 < \phi_1 < 10$ with weights depending on $\phi_1$. The Gaussian fit with 640 stars and sigma=9' is shown in red.

FIG. 3.—Color-color diagram of the stream. The metallicity according to the Equation 4 from Ivezić et al. (2008) is $[\text{Fe/H}] = -1.9 \pm 0.1$.

halo stars, and therefore the Ivezić et al. (2008) calibration is correct). We derive that $[\text{Fe/H}]_{\text{phot}} = -1.9 \pm 0.1$.

To derive the metallicity age and distance in a more
Fig. 1.— Color magnitude diagrams of the stream in different filters and the Girardi isochrones for $t=7$ Gyr, $[Fe/H]=-1.3$, distance=8.5 kpc.

Fig. 5.— The stellar population fit to the color magnitude diagrams in u,g,r,i,z bands of the stream. The panels show the 90%, 99%, 99.9% confidence contours of the parameters. Filled circles shows the location of the best goodness of fit point.

Fig. 6.— Distance variation along the stream. The CMD diagrams of the stream for 2 different parts of the stream (left panel $-10 < \phi_1 < -20$, right panel $-10 < \phi_1 < 10$). The isochrones for the the best fit model $Z=0.001$, age=9 Gyr were shifted to the distance of 8.5 kpc on the left panel and to 1 kpc on the right panel. Slight distance variation is visible. The stellar population shown on the right panel is clearly located at further distances the the stellar population from the left panel.

B1.0 (Monet et al. 2003) data by Munn et al. (2004). We use the proper motions from the SDSS DR7 data release, since earlier versions of the proper motions contained systematic errors (Munn in prep... ? ).

Figure 2 shows how the stream stars are distributed and allows for the statistical proper motion determination. To see that signal We can use the method which is similar to the method used to derive the Hess diagram of the stream. We select a subset of stars with proper motions within a box $\mu_u, \mu_v, \mu_o, \mu_r, \mu_i, \mu_z < 10$. Then for the subset of stars with these proper motions we can construct the profile of stars similar to the profile shown on Fig. 2, which can be fitted by constant plus a gaussian model (Equation 1). Therefore for each proper motion bin we derive $N_{\text{stream}}$ number of stream stars. To remove a significant contamination from the stars not belonging to the stream, instead of using the stars lying inside a color magnitude box, we use the isochrone filter derived from isochrones from Sec. 2. We create the model
Fig. 17.— The $\chi^2$ surface of the orbit fit for the 3 component potential (Eq. 9, 8 and 7) with different disk masses $M_d$, halo circular velocities $v_{halo}$ and halo flattenings $q_{halo}$. The fit was marginalized over the circular velocity of the halo $v_{halo}$. The contours show the $1\sigma$, $2\sigma$, $3\sigma$ confidence limits. The $\chi^2$ values on that plot take into account the prior on the circular velocity of the LSR from Ghez et al. (2008) (229 ± 18 km/s).
cold streams: kinematics

- working in the *cold limit*, velocity along the stream plus curvature of the stream determines the local *acceleration vector*
- must assume time-independence or something equally restrictive
- we have also done this for the Grillmair stream
cold streams: observational challenges

- at the present day, streams are only detected at low significance
- no star on the sky is associated with the stream at high significance
- no stellar position or velocity is measured at high significance
- determine configuration-space and velocity-space shape of stream through statistical foreground–background modeling
- each stream is given individual love and attention
- this will—or must—change with Gaia
what if the halo is \textit{filled} with streams?

- many streams could produce the full acceleration field of the Galaxy
- integrate for the potential
- potentially extremely powerful, but
  - is the halo dense with streams?
  - can we avoid treating each stream individually?
  - cannot measure and integrate the acceleration for a time-dependent potential
  - cannot detect details of massive substructure with this (alone)
two-star streams

- generically, stars are born in small groups and binaries
- many of these get disrupted tidally
- the Galaxy should be filled with two-star “streams”
- ought to be detectable in the two-point function
  - proposed by Tremaine
  - predictable morphology of the two-point function in phase-space
  - never been done, but straightforward...
phase-space stream structure

- (recall: the “roulette” approach was to find the potential that \textit{minimizes} structure in angle-space)
- the “cold streams” approach is to find the potential that \textit{maximizes} structure in \textit{action-space}
  - in detail, maximize structure in the 5-d space of actions and the two angles perpendicular to the tidal-disruption direction in angle space.
potentials are never (naively) integrable!

- Milky Way is filled with substructure
- Milky Way is accreting satellites and mass continuously
- no time-independence
- no axisymmetry
- no known symmetries of any kind
recap

- *first* we assumed integrable potential and mixed angles
- *second* we assumed integrable potential but unmixed angles
- *now* we will assume *neither*
stream perturbations

- imagine that the galaxy is close to integrable, but has compact, orbiting massive substructures
- cold streams are coherent objects that get perturbed by close passages
- the perturbation is long-lived and coherent
  - remembers things about its origin
  - but with near degeneracies (degeneracies in practice)
observational degeneracies

- until we can measure all stream-star velocities \((3d)\) to part-in-thousand:
- any observed perturbation gives you impact parameter \(b\)

\[
b
\]

and an amplitude

\[
\frac{2 G M T}{b v},
\]

where \(T\) is the time since the event, and \(v\) is the speed of the perturber

- you can’t break the degeneracy without incredibly precise velocities
imaging substructure

- imagine a massive substructure passing through a tangle of cold streams
- each stream in its past history has a memory
- each favors a degeneracy space of perturber masses and orbits
- gravitational discovery of perturbers
- gravitational confirmation of dark-matter annihilation signals
will we have lots of streams?

▶ yes!
▶ in SDSS there are a handful of cold streams each containing hundreds of stars
▶ SDSS is a 2.5d map of the stellar distribution
▶ Gaia is a 5.5d map
▶ streams with only tens of stars will be easily detectable
the general problem

- can we perform inference if we *can’t* assume phase mixing or integrability?
- yes, of course! The keys:
  - stars form in cold clumps in phase space
  - co-eval stars (ought to) show chemical similarities
  - Milky Way forms by gravitational collapse from a homogeneous neighborhood
  - fit dark-matter initial conditions (including phases) and the birthrate (as a function of phase-space through time)
- (no-one said it would be easy)
- relates to other *comprehensive modeling* problems in astrophysics and elsewhere
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when phases aren’t mixed, we are using anisotropies that develop in phase space to perform dynamical inference
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when phases aren’t mixed and potentials aren’t integrable, phase-space structure is still highly informative
  ▶ perturbed cold stellar streams
  ▶ comprehensive modeling from \(t = 0\)