RELIC ABUNDANCES

Now we are ready to discuss the abundance ~~of~~ expected for particles with different masses and interactions that decoupled (or "freezed out", as it is also described) early in the universe. ~~Such~~ Such abundances are known as "relic abundances", a left over from the early thermal history of the universe. These calculations are an essential tool to understand the properties of dark matter, calculating relic abundances places important constraints on the properties (interactions, masses) of hypothetical dark matter candidates. As usual, we'll distinguish two cases: "hot relics" (decouple when relativistic, though today they are NR) and "cold relics" (which freeze out when NR). Generically, the cross section necessary to have an interesting abundance today (to act as dark matter) points to weak-interaction cross sections, thus why these hypothetical particles are usually referred to as WIMP's (Weakly Interacting Massive Particles).

Hot Relics

Let's consider a WIMP (e.g. a massive neutrino) with mass $m \ll T_{dec}$, relativistic when it freezes out. As long as the mass is large enough compared to the present temperature ($m \gtrsim T_0 \sim 2K \sim 1.7 \times 10^{-4}$ eV) however, these particles will become non-relativistic at some point in the past. We shall assume these particles are stable (i.e. their lifetime, if not ∞, is larger than the age of the universe).

The relic abundance is, as we discussed

$$n = \left(\frac{3}{4}\right) g \frac{\zeta(3)}{\pi^2} T^3 \qquad \rho \simeq m\, n$$

and I assumed particles to be fermions, thus why factor of 3/4.

Let's assume this neutrino has std weak interactions, and decouples when $T \simeq 1$ MeV and thus its temperature today is $T \sim 1.9\,°K$. Then we can calculate its contribution to Ω_m today as:

$$\Omega_\nu h^2 = \frac{\rho_\nu h^2}{\rho_{crit}} = \frac{m\, n}{\rho_{crit}/h^2} \underset{\substack{\uparrow\\ \text{everything}\\ \text{in GeV's}}}{=} \frac{m}{1\,\text{GeV}} \frac{\frac{3}{4}\times\frac{7}{8}\times 2 \times \frac{1.2}{\pi^2}}{8\times 10^{-47}} \left(\frac{1.9}{1.16\times 10^{13}}\right)^3$$

$$\Rightarrow \quad \Omega_\nu h^2 \simeq \left(\frac{m}{100\,\text{eV}}\right)$$

from this we see that since $\Omega_m h^2 < 1$ we can set a limit $m < 100\,\text{eV}$ (or one can be more accurate and require $\Omega_m h^2 \sim 0.15 \pm 0.05$)

[Note: In assuming that $n \sim T^3$ above we used the fact that $f = f_{eq} = \text{const}$ equal to the relativistic f_{eq}, even though today is NR, as we discussed last class]

In general, the abundance will depend on when the species decouples, we will explore a bit more in the homework...

Cold Relics ($m \gg T_{dec}$)

These are species that decouple while already non-relativistic. We have two cases depending on how their mass m compares to the Z-mass (mediator of weak int.)

i) $m \ll m_Z$

Since it is NR, its typical energy is m, and in this case the cross-section is simply $\sigma \sim G_F^2 m^2$

@ $T = T_D$ we have (let's use "D" instead of "dec")

$$\underbrace{g\left(\frac{m T_D}{2\pi}\right)^{3/2} \exp\left[-m/T_D\right]}_{n_D} \underbrace{G_F^2 m^2}_{\sigma} = \underbrace{1.66\, \frac{T_D^2}{m_{pl}}\, g_*^{D\,1/2}}_{H}$$

where I assumed $v \sim 1$, $\mu = 0$.

Then the abundance @ decoupling is simply, from this condition

$$n_D \simeq \frac{1.66\, T_D^2}{m_{pl}}\, g_*^{D\,1/2} \left(G_F^2 m^2\right)^{-1}$$

The abundance today is then

$$n_0 = n_D \left(\frac{a_D}{a_0}\right)^3 \underset{\substack{\uparrow\\ \text{conservation}\\ \text{of entropy}}}{=} n_D \frac{(g_*^S)^0}{(g_*^S)^D} \frac{T_0^3}{T_D^3}$$

Then we ~~can~~ calculate the contribution of these wimps (w) to $\Omega_m h^2$ today: 3

$$\Omega_w h^2 = \frac{2\times n_0 m}{8\times 10^{-47}\,\text{GeV}^4} = \frac{2\times 1.66\left(\frac{2.7}{1.16\times 10^{13}\,\text{GeV}^{-1}}\right)^3 3.91\, g_*^{D\,1/2}}{8\times 10^{-47}\,\text{GeV}^4\; 1.22\times 10^{19}\,\text{GeV}\,[1.16\times 10^{-5}\,\text{GeV}^{-2}]^2 (g_*^s)^D}\frac{m}{T_D}\frac{1}{m^2}$$

wimp and anti wimp

$$\Rightarrow \quad \Omega_w h^2 \simeq 1\, \frac{g_*^{D\,1/2}}{(g_*^s)^D}\left(\frac{m}{T_D}\right)\left(\frac{1\,\text{GeV}}{m}\right)^2$$

Now we have to figure out m/T_D from the decoupling condition above. Taking logs:

$$-\frac{m}{T_D} = \underbrace{\ln\left[\frac{1.66\,(2\pi)^{3/2}}{m_{Pl}\;G_F^2\;1\,\text{GeV}^3}\right]}_{-15.2} - 3\ln\left(\frac{m}{1\,\text{GeV}}\right) + \frac{1}{2}\ln\left(\frac{T_D}{m}\right) + \ln\left(\frac{g_*^{D\,1/2}}{g}\right)$$

$$\Rightarrow \quad \boxed{\frac{m}{T_D} = 15.2 + 3\ln\left(\frac{m}{1\,\text{GeV}}\right) - \frac{1}{2}\ln\left(\frac{T_D}{m}\right) - \ln\left(\frac{g_*^{D\,1/2}}{g}\right)}$$

This is an implicit equation, which we will solve roughly by taking advantage of slow log's, by iteration. Since $m \ll m_Z \sim 100$ GeV, let's take $m \sim 1$ GeV, then up to log's $\frac{m}{T_D} \simeq 15.2 \Rightarrow T_D = \frac{1\,\text{GeV}}{15.2} \sim 70$ MeV

$\Rightarrow g_*^D \sim 20$ and assuming $g \simeq 2$ we have for the log ~~terms~~

$$\left.\begin{aligned} -\frac{1}{2}\ln\left(\frac{T_D}{m}\right) &= +1.36 \\ -\ln\left(\frac{\sqrt{20}}{2}\right) &= -0.8 \end{aligned}\right\} \Rightarrow \quad \frac{m}{T_D} \simeq 15.8 + 3\ln\left(\frac{m}{1\,\text{GeV}}\right)$$

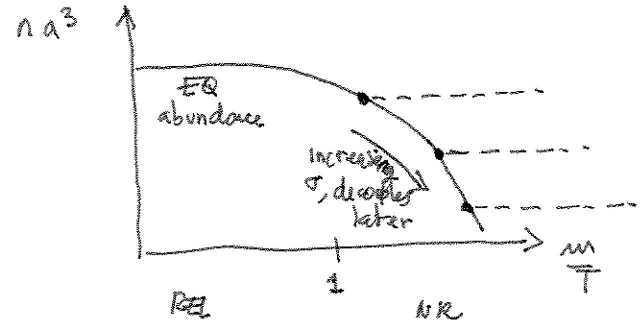
this is to remember we used $m \simeq 1$ GeV

$$\Rightarrow \quad \boxed{\Omega_w h^2 \simeq \frac{15.8}{\sqrt{20}}\left(\frac{1\,\text{GeV}}{m}\right)^2 \sim 3\left(\frac{1\,\text{GeV}}{m}\right)^2} \quad (m \ll m_Z)$$

$(g_*^s)^D \sim (g_*)^D \sim 20$

Note that $\Omega_w h^2$ decreases with increasing m, unlike for the case we discussed before for hot relics. The reason is that for $m \ll m_Z$, the cross section increases with m ($\sigma \sim G_F^2 m^2$), then larger m means that it will stay in equilibrium longer and thus gets equilibrium abundance Boltzmann suppressed:

$$(n a^3)_{EQ} \sim (n T^{-3})_{EQ} \sim \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



So, the stronger it interacts, the smaller its abundance today.

ii) $m \gg M_Z$ $\Rightarrow$ $\sigma \sim \frac{\lambda^4}{m^2}$ ~~for~~ the NR case. We take $\lambda^4 \sim 10^{-3}$ as typical value.

Then we have as before (just changing cross section)

$$n_D = \frac{1.66\, T_D^2}{m_{pl}}\, g_*^{D\,1/2}\, \frac{m^2}{\lambda^4}$$

$$\Rightarrow \Omega_w h^2 = \frac{2m}{8\times 10^{-47}\,GeV^4}\; \frac{1.66\, \cancel{T_D^2}}{m_{pl}}\, g_*^{D\,1/2}\, \frac{m^2}{\lambda^4}\; \frac{(g_*^s)^0}{(g_*^s)^D}\; \frac{T_0^3}{T_D^{\cancel{3}}}$$

$$\Rightarrow \Omega_w h^2 = \left(\frac{m}{1\,TeV}\right)^2 \left(\frac{m}{T_D}\right) \times 2\, \frac{1.66\, g_*^{D\,1/2}\, (g_*^s)^0\, T_0^3\, (1\,TeV)^2}{4\times 10^{-47}\,GeV^4\, m_{pl}\, \lambda^4\, (g_*^s)^D}$$

Again, to get (m/T_D) we impose $\Gamma = H$:

$$g\left(\frac{m T_D}{2\pi}\right)^{3/2} e^{-m/T_D} = 1.66\, \frac{T_D^2}{m_{pl}}\, g_*^{D\,1/2}\, \frac{m^2}{\lambda^4}$$

$$\Rightarrow e^{m/T_D} = \frac{g}{(2\pi)^{3/2}}\; \frac{m_{pl}\,\lambda^4}{1.66\, g_*^{D\,1/2}}\; \frac{1}{\sqrt{m T_D}}$$

$$\Rightarrow \frac{m}{T_D} = \ln\left[\frac{g\,\lambda^4}{(2\pi)^{3/2}\, 1.66\, g_*^{D\,1/2}}\right] + \underbrace{\ln\left(\frac{m_{pl}}{TeV}\right)}_{37.5} + \ln\left(\frac{TeV}{m}\right) + \frac{1}{2}\ln\left(\frac{m}{T_D}\right)$$

Now:

$$\ln\left[\frac{g\,\lambda^4}{(2\pi)^{3/2}\ 1.66\ g_*^{D\,1/2}}\right] \sim \ln\left[\frac{2\times10^{-3}}{(2\pi)^{3/2}\ 1.66\times10}\right] \simeq 11.7$$

$g_*^D \sim 100$ at high energies

$$\Rightarrow \quad \frac{m}{T_D} \simeq 25.75 + \ln\left(\frac{1\,\mathrm{TeV}}{m}\right) + \frac{1}{2}\ln\left(\frac{m}{T_D}\right)$$

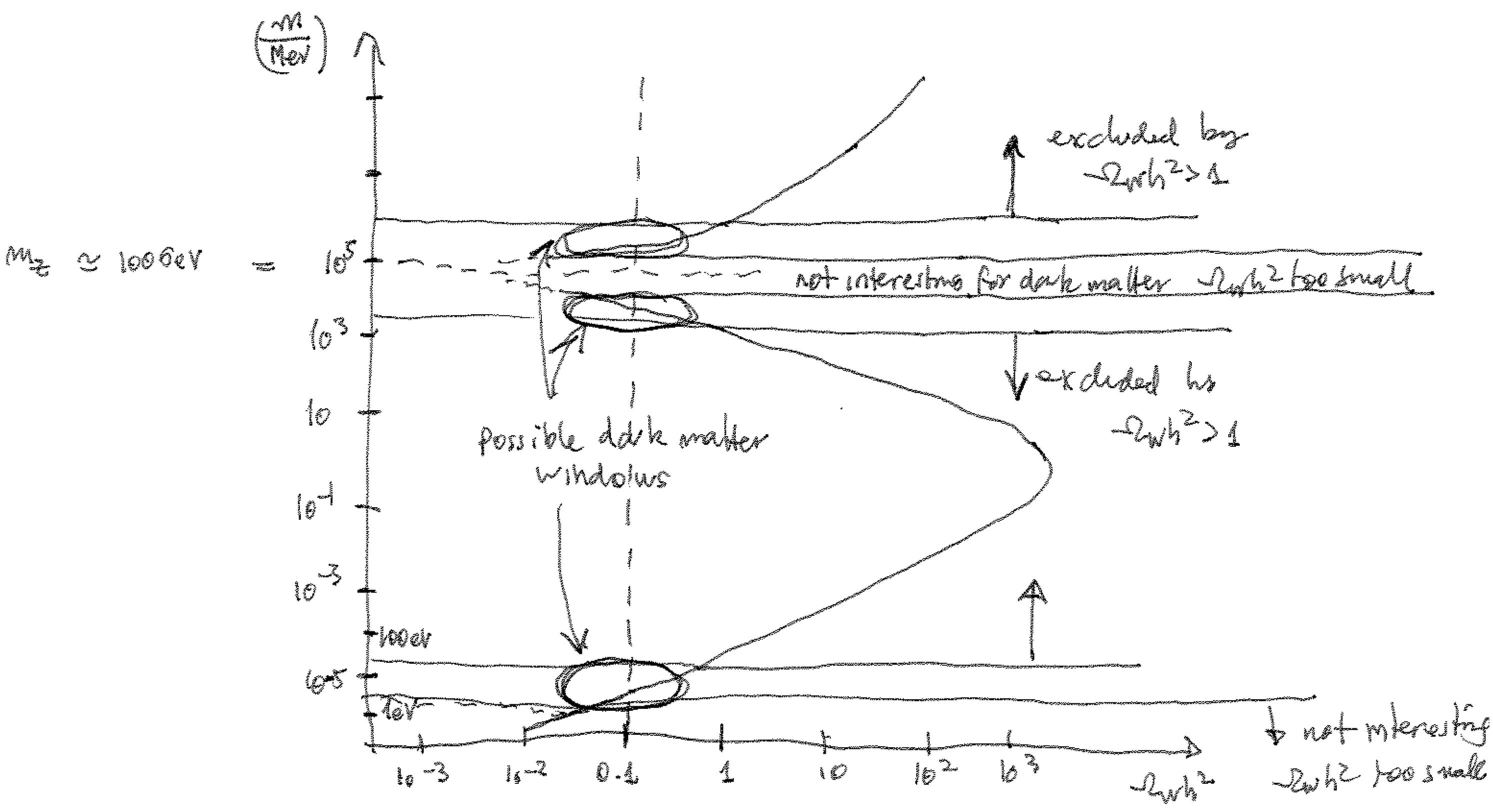
Iterating, the log factor corrects this to $\frac{m}{T_D} \simeq 27.4 + \ln\left(\frac{1\,\mathrm{TeV}}{m}\right)$

$$\Rightarrow \quad \Omega_w h^2 \simeq \left(\frac{m}{1\,\mathrm{TeV}}\right)^2 \times \underbrace{\frac{27.4 \times 2 \times 1.66 \times 10 \times 3.91\ \left(\frac{2.7}{1.16\times10^{13}}\right)^3 \cancel{\mathrm{GeV}^5}\times10^6}{4\times10^{-47}\ \cancel{\mathrm{GeV}^4}\ 2\times10^{2}\ \cancel{\mathrm{GeV}}\ 10^{-3}\times 100}}_{\simeq\,0.6\,\sim\,1}$$

$$\Rightarrow \quad \boxed{\Omega_w h^2 \sim \left(\frac{m}{1\,\mathrm{TeV}}\right)^2} \qquad (m \gg m_Z)$$

Note here the different dependence on m due to the change of behavior of the cross section.

We can summarize all these results by the following plot (see fig 3.1 in Padmanabhan)



The main candidates for non-baryonic dark matter are roughly:

Relic "particle"	mass	Origin (t, T)	Abundance $[cm^{-3}]$
Axion (non-thermally produced)	10^{-5} eV	10^{-30} sec, 10^{12} GeV	10^{9}
Light ν	30 eV	1 sec, 1 MeV	10^{2}
Photino - Gravitino	keV	10^{-4} sec, 100 MeV	10
Heavy ν, neutralino axino, photino, sneutrino	GeV	10^{-3} sec, 10 MeV	10^{-5}
Magnetic monopoles	10^{16} GeV	10^{-34} sec, 10^{14} GeV	10^{-21}
Primordial blackholes	$\gtrsim 10^{15}$ g	$\gtrsim 10^{-12}$ sec, $\lesssim 10^{3}$ GeV	$\lesssim 10^{-44}$