BIG-BANG NUCLEOSYNTHESIS (BBN)

As the universe cools down below $T \sim 0.2$ GeV where the quark-hadron phase transition takes place, the universe is filled by γ's, ν's and the familiar $n, p, e, \ldots$, etc. As nuclear binding energies are approached, one naively expects light nuclei to start forming.

Recall for a nucleus $^{A}X_{Z}$ with Z protons and N neutrons with atomic mass $A = Z + N$ the binding energy is given by $B_A = Z m_p + (A-Z) m_N - m_A$ where m_p is the proton mass, m_N the neutron mass. For the first few light nuclei we have:

			Z	g_A
Deuterium	^2H	$B = 2.22$ MeV	1	3
Tritium	^3H	6.92 MeV	1	2
Helium 3	^3He	7.72 MeV	2	2
Helium 4	^4He	28.3 MeV	2	1
Carbon	^{12}C	92.2 MeV	6	1

Therefore, naively one expects nucleosynthesis to begin at temperatures $T \sim 1-30$ MeV : this is wrong! The reason this is not the case has to do with the fact that there are a huge number of photons compared to baryons (protons, neutrons, $m_B \simeq m_N \simeq m_p$). Indeed,

$$\eta \equiv \frac{n_B}{n_\gamma} = 2.68 \times 10^{-8} \, \Omega_B h^2 \qquad (\Omega_B h^2 \sim 0.02)$$

Therefore, since there are so many photons per baryon, there are always photons of energy high enough to break nuclei up in the tail of the Planck spectrum. In other words, one has to go to temperatures well below 1-30 MeV before these abundant high energy photons get suppressed enough to allow nuclei to form.

In order to see this formally, let's assume everything is in thermal equilibrium, at these temperatures things are non-relativistic so the abundance of element A is given by,

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_A - \mu_A)}{T}\right]$$

For protons and neutrons we have

$$\begin{cases} n_p \simeq 2 \left(\frac{m_B T}{2\pi}\right)^{3/2} \exp\left[-\frac{m_p - \mu_p}{T}\right] \\ n_N \simeq 2 \left(\frac{m_B T}{2\pi}\right)^{3/2} \exp\left[-\frac{m_N - \mu_N}{T}\right] \end{cases}$$

where we have approximated $m_p \simeq m_N \simeq m_B$ (baryon mass) in prefactor, but kept $m_p \neq m_N$ in the exponentials. Recall that

$$\begin{cases} Q \equiv m_N - m_p = 1.293 \text{ MeV} \\ m_B \simeq 1 \text{ GeV} \end{cases}$$

In chemical equilibrium, $Z\,p + (A-Z)\,N \Leftrightarrow A + \gamma$, implies

$$\mu_A = Z\mu_p + (A-Z)\mu_N$$

$$\Rightarrow e^{\mu_A/T} = e^{[Z\mu_p + (A-Z)\mu_N]/T} = n_p^Z\, n_N^{A-Z} \left(\frac{2\pi}{m_B T}\right)^{\frac{3A}{2}} 2^{-A}\, e^{[Zm_p + (A-Z)m_N]/T}$$

$$\Rightarrow n_A = g_A\, A^{3/2}\, 2^{-A} \left(\frac{2\pi}{m_B T}\right)^{\frac{3}{2}(A-1)} n_p^Z\, n_N^{A-Z} \exp\left[\frac{B_A}{T}\right]$$

Since particle number densities evolve as $\sim a^{-3}$ (for constant number per comoving volume) it is useful to normalize abundance by that of the total number density of baryons n_B quoted above. In particular, it is useful to work with the <u>mass</u> fractions X_A:

$$X_A \equiv A\,\frac{n_A}{n_B}$$

where for $i = 1, \ldots, N$ species we have

$$\sum_{i=1}^{N} X_i = 1 \quad , \quad n_B \equiv n_p + n_N + \sum_{i=1}^{N} (A\, n_A)_i$$

Then we have: $\left(\eta \equiv \frac{n_B}{n_\gamma} , \quad n_\gamma = 2 \frac{\zeta(3)}{\pi^2} T^3 \right)$

$$\begin{cases} n_A = \frac{n_B}{A} X_A = \eta\, n_\gamma \frac{X_A}{A} \\ n_p = n_B X_p = \eta\, n_\gamma X_p \\ n_N = n_B X_N = \eta\, n_\gamma X_N \end{cases}$$

$$\Rightarrow \boxed{X_A = \frac{A\, n_A}{\eta\, n_\gamma} = \left[g_A A^{5/2} \zeta(3)^{A-1} \pi^{1-A/2} 2^{(3A-5)/2} \right] \left(\frac{T}{m_B} \right)^{\frac{3(A-1)}{2}} \eta^{A-1} X_p^{Z} X_N^{A-Z} e^{B_A/T}}$$

Since $\eta \sim 10^{-9} \ll 1$ we need $T \ll B_A$ to have a non-negligible abundance in equilibrium. A quick estimate we can obtain by setting $X_p \sim X_N \sim 1$ and then $X_A \sim 1$ when (setting $[\,] \simeq 1$)

$$T_{NUC} \sim \frac{B_A / (A-1)}{\ln(\eta^{-1}) + \frac{3}{2} \ln \frac{m_B}{T}}$$

which gives: for
$$\begin{cases} {}^2H & : 0.07 \text{ MeV} \\ {}^3He & 0.11 \text{ MeV} \\ {}^4He & 0.28 \text{ MeV} \\ {}^{12}C & 0.25 \text{ MeV} \end{cases}$$

compare this with B_A numbers given before!

Therefore, BBN happens much later than naively expected from just B_A arguments, one <u>must</u> take into account the large number of photons per baryon. [This is very different in stars, where $\eta \sim 10^{+2}$]

So far the discussion has assumed thermal equilibrium. It turns out, however that thermal equilibrium <u>does not</u> persist down to $T \sim 0.28 - 0.07$ MeV. Therefore, one must go beyond equilibrium to discuss BBN abundances.

There are two crucial general type of reactions that determine the abundance of elements

i) $\frac{n}{p}$ determined by weak interaction rates

ii) nuclear reaction rates

We have to study when these reactions go out of equilibrium.

i) protons and neutrons interact by

$$\begin{cases} n \leftrightarrow p + e^- + \bar{\nu} \\ \nu + n \leftrightarrow p + e^- \\ e^+ + n \leftrightarrow p + \bar{\nu} \end{cases}$$

in equilibrium ($\Gamma > H$), we have $\mu_N + \mu_\nu = \mu_p + \mu_e \Rightarrow \mu_N - \mu_p = \mu_e - \mu_\nu \simeq 0$ since $\mu_e \sim \mu_\nu \sim 0$. Then

neutron to proton ratio: $$\left(\frac{n}{p}\right)_{EQ} \equiv \left(\frac{n_N}{n_p}\right)_{EQ} = \left(\frac{X_N}{X_p}\right)_{EQ} = \exp\left[-\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T}\right] \simeq \exp\left(-Q/T\right)$$

where recall that $Q \equiv m_N - m_p \sim 1.3\ \text{MeV}$. As the universe cools below $T \sim 1.3\ \text{MeV}$ protons are favoured over neutrons. To see when this equilibrium neutron to proton ratio is no longer mantained as T goes down one must calculate the interaction rate from weak interactions. The result of this is that

$$\Gamma \sim \tau_n^{-1} f(T) \sim G_F^2 T^5$$

where the neutron lifetime $\tau_n \sim 915.4$ sec $\sim G_F^{-2}$

the half-life is $\tau = \tau_n \times \ln 2 \sim 10.5$ min

Putting numbers:

$$\frac{\Gamma}{H} \simeq \left(\frac{T}{0.8\ \text{MeV}}\right)^3$$

So when $T \gtrsim 0.8$ MeV $\frac{n}{p} = \left(\frac{n}{p}\right)_{EQ}$, which at $T \gg 1$ MeV gives essentially $\frac{n}{p} \simeq 1$ and $X_N = X_p$.

When temperature drops below $T \simeq 0.8$ MeV the neutron to proton ratio freezes out ("decouples") and ~~always~~ would stay constant. However, since the neutron is unstable by beta decay its abundance does not stay actually constant but decays with a half-life of $\tau \sim 10$ min. So from 0.8 MeV down to T where nucleosynthesis starts the abundance of n/p will be somewhat smaller than the $(n/p)_{freeze-out}$ value.

ii) The nuclear rates can be shown to be in equilibrium at temperatures of 1 MeV, so at this temperature we have n, p, γ's, ν's and A's in equilibrium, although the X_A is negligible here due to η being so small as we discussed. Let's follow what happens as T decreases from 1 MeV all the way to when nucleosynthesis happens:

- As T drops below 1 MeV, many interesting things take place:

  1) ν's decouple at $T \sim 1$ MeV, as we discussed before
  2) a bit later @ $T \sim 0.5$ MeV, e^+e^- annihilate transferring their entropy to the photons. This also was discussed already.
  3) when $T \sim 0.8$ MeV, the n/p freezes out with a value

$$\left(\frac{n}{p}\right)_{\text{freeze-out}} = \exp\left(-\frac{Q}{T_F}\right) \simeq \exp\left(-\frac{1.3}{0.8}\right) \simeq \frac{1}{6}$$

At this time nuclear reactions are still in equilibrium, however abundances are really small due to small value of η:

$X_N \sim \frac{1}{7}$ $X_p \sim \frac{6}{7}$ but $X_2 \sim 10^{-12}$ (Deuterium)

$X_3 \sim 10^{-23}$ (Tritium)

$X_4 \sim 10^{-28}$ (^{4}He)

- As T drops to $T \sim 0.3 - 0.1$ MeV ($t \simeq 1$-3 minutes)

one expects nucleosynthesis to take place – let's see what happens–
By this time some neutrons have already decayed, then since
$\exp(-3\text{min}/10\text{min}) \simeq 0.8$, the freeze-out ratio gets slightly
changed:

$$\left(\frac{n}{p}\right)_{NUC} \sim \underbrace{\left(\frac{n}{p}\right)_{\text{freeze-out}}}_{\simeq 1/6} \times 0.8 \simeq \frac{1}{7}$$

[Recall that if we were at equilibrium this abundance would be quite
smaller, $(n/p)_{EQ} \simeq \exp\left(-\frac{Q}{T}\right) = \exp\left(-\frac{1.3}{0.3}\right) \simeq \frac{1}{76}$!!]

From the equilibrium calculation done before we expect at $T \sim 0.28$ MeV
that X_4 becomes of order unity.
However, the fastest way to
create ^4He is through deuterium,
and then tritium and Helium 3:

[Note: here we assume EQ for nuclear reaction rates, relevant for nuclei. The weak interaction rates already dropped below H and that's why n/p freezed out. See distinction between i) and ii) above.]

$$^2H + {}^2H \leftrightarrow {}^3H + p$$
$$^2H + {}^2H \leftrightarrow {}^3He + n$$
$$^3H + {}^2H \leftrightarrow {}^4He + n$$
$$^3He + {}^2H \leftrightarrow {}^4He + p$$

[Unlike stars, where densities are large, here only 2-body reactions are important]

Now, since deuterium is formed by

$$n + p \longleftrightarrow {}^2H + \gamma$$

and it is so weakly bound ($B = 2.2$ MeV), the large number of γ's around
(low η) means that one has to wait until $T \simeq 0.1$ MeV before enough ^2H
can stay bound against photodissociation by γ's, similarly for ^3H
and ^3He, see estimates of T_{NUC} above. This is sometimes referred to

as "the deuterium bottleneck" (though in reality is due to η small, and happens also for tritium and ^3He)

- When $T \simeq 0.1$ MeV, the reactions above are fast enough and the abundance of required elements is close to unity so things proceede to create an equilibrium abundance of ^4He, the most bound nucleus of the first few light elements. In fact, it is a reasonably good approximation that essentially all neutrons end up inside ^4He, then we can roughly estimate the mass fraction of ^4He by

$$\boxed{X_4 = \frac{4\, n_{4He}}{n_B} \simeq \frac{4\,(n_N/2)}{n_N + n_P} = \frac{2\,(n/p)_{NUC}}{1+(n/p)_{NUC}} \underset{\uparrow}{\simeq} \frac{1}{4}}$$

$$\left(\frac{n}{p}\right)_{NUC} \sim \frac{1}{7}$$

This is an amazing result, simple arguments tell us that about 25% of Helium is what one should expect coming from the Early Universe. More elaborate calculations give a very similar result.

- As reactions ~~proceed~~ ^2H, ^3H, ^3He get depleted and since the rates a proportional to number densities ($\Gamma \sim n_A$) and number densities get depleted the nuclear interaction rates go down significantly and freeze out, leaving a residual fraction of deuterium and ^3He. Since $n_A \sim X_A\, \eta\, n_\gamma$, these abundances depend on η sensitively. On the other hand, ^4He depends mostly on the value of (n/p) at $T \simeq 0.1$ MeV.

- What happens with heavier elements? They are suppressed for the following reasons

i) By the time ^2H is available to form ^4He, the Coulomb barrier becomes large compared to energy T:

Coulomb barrier penetration $\sim \exp\left[-2\,\bar{A}^{1/3}\,(Z_1 Z_2)^{2/3}\left(\frac{T}{MeV}\right)^{-1/3}\right]$

with $\bar{A} \equiv \frac{A_1 A_2}{A_1 + A_2}$, this is very significant for higher A's,

ii) There are no tightly bound elements with $A = 5, 8$

iii) The density is low enough that the triple α reaction $3\,{}^4He \to {}^{12}C$ is strongly suppressed (this works in stars though)

Some traces of ${}^7Li$ and ${}^7Be$ are produced through ${}^4He + {}^3H \to n + {}^7Li$ and ${}^4He + {}^3He \to \gamma + {}^7Be$, leading to

$$\frac{{}^7Li}{H} \sim 10^{-9} - 10^{-10} \qquad \frac{{}^7Be}{H} \sim 10^{-11}$$

The residual fraction of deuterium and ${}^3He$ is

$$\frac{D}{H} \sim \frac{{}^3He}{H} \sim 10^{-4} - 10^{-5}$$

these amounts, left "unburnt" are inversely proportional to η by the arguments above

[Show BBN ~~[illegible]~~ results]

Summarizing, the abundance of light elements is sensitive to the baryon to photon ratio η, and two physical parameters $g_*(T \sim 1\,MeV)$ and the half-life of the neutron. We will explore these dependencies in the homework.

From the observational point of view, the situation is challenging, as one must be able to take into account post-processing of the primordial abundance by stars, cosmic ray spallation, etc.

For example, since deuterium is so weakly bound, it is only destroyed, therefore ${}^2H_{TODAY} < ({}^2H)_{primordial}$, thus ~~the~~ observations put a lower bound on the primordial abundance $\Rightarrow$ an upper bound on η $\Rightarrow$ upper bound on Ω_B.

Ways in which deuterium is observed today includes: D molecules in Jupiter atmosphere, meteorites and solar wind, UV absorption lines in ISM with HST, $Ly\text{-}\alpha$ absorption in high-z quasars. This leads roughly to
$$\left.\frac{D}{H}\right]_{primordial} \gtrsim 2\times10^{-5} \quad\Rightarrow\quad \Omega_B h^2 \lesssim 0.03$$

This is important because it clearly shows that baryons cannot have large enough densities to account for dark matter.

Though deuterium is destroyed, it turns into ^3He, and stars do produce ^3He. Then the sum $D+{}^3He$ measured is larger than primordial (also, ^3He is more difficult to destroyed, without producing lots of ^4He). From this we can obtain an upper bound: $\frac{D+{}^3He}{H} \lesssim 1.1\times10^{-4} \Rightarrow \Omega_b h^2 \gtrsim 0.007$

As for ^4He, stars are net producers, the production being correlated with that of heavier elements ("metals" in astronomer's jargon). Although it is significantly easier to measure ^4He abundances due to their larger numbers, it does not constrain η significantly due to its rather flat dependence on η. From studying H II regions (by looking at the recombination line of He and H), people extrapolate to zero metalicity (primordial) to get roughly

^{4}Helium abundance: $Y_p \simeq 0.232 \pm 0.003\,(\text{stat}) \; {}^{+0.016}_{-0.012}\,(\text{syst.})$

All these results are consistent with $10^{-10} \lesssim \eta \lesssim 10^{-9}$ and, further, the dependence on g_* can be used to set a limit on the number of neutrino families (relativistic @ NUC), giving $N_\nu \simeq 3$. This was obtained before direct experiments at accelerators!

We shall see later that from CMB anisotropies $\Omega_b h^2 \simeq 0.024 \pm 0.003$, $\eta = 6.5^{+0.4}_{-0.3}\times10^{-10}$

Brief Description of Nucleosynthesis in stars

In a typical young star like ~~the~~ sun, the bulk of energy is produced by converting ~~the~~ protons into ^4He nuclei, much the same way as in the early universe (though under very different physical conditions). This fusion process converts about 0.7% of mass into energy, the most efficient energy production mechanism known.

Although this process is exothermic, the probability of 4 protons being fused together into ^4He is too small, the most likely path being through deuterium as we discussed in BBN:

$$H + H \rightarrow \underset{^2H}{D} + e^+ + \nu_e$$

$$H + {}^2H \rightarrow {}^3He + \gamma$$

$${}^3He + {}^3He \rightarrow {}^4He + H + H$$

Since we get the two protons back, the net outcome of this process is to convert 4 protons into a ^4He (plus ~28 MeV of energy). This is known as the PPI chain (for proton-proton, I because there are many pp-chains)

Once there is enough ^4He there is a second stage (PP II):

$${}^3He + {}^4He \rightarrow {}^7Be + \gamma$$

$$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$$

$$H + {}^7Li \rightarrow {}^4He + {}^4He$$

This again converts basically 4 protons into ^4He (note you get one ^4He back).

Another route for hydrogen burning is the PP III chain,

$$H + {}^7Be \rightarrow {}^8B + \gamma$$

$${}^8B \rightarrow {}^8Be + e^+ + \nu_e$$

$${}^8Be \rightarrow {}^4He + {}^4He$$

The interest here is the high energy (~7 MeV) of the neutrino ~~released~~ released in the inverse β decay of ^8B. These ν's are ~~used to study~~ detected in solar neutrinos ~~ex~~ experiments in chlorine detectors.

When most hydrogen is converted into ^4He, the production of fusion energy drops significantly and the star cools down due to radiation. As T drops, thermal pressure is reduced and the star contracts again until T increases above H burning, upto 10^8 K when ^4He nuclear reactions can take place, converting 3 ^4He nuclei into ^{12}C:

$$^4He + {}^4He \to {}^8Be$$
$$^4He + {}^8Be \to {}^{12}C + \gamma$$

When ^{12}C abundance goes up, ^4He capture by ^{12}C becomes important, leading to oxigen,

$$^4He + {}^{12}C \to {}^{16}O + \gamma$$

(similarly, by α-particle, ^4He, capture one forms ^{20}Ne and ^{24}Mg)

When all ^4He is used up, there is another stage of gravitational contraction and heating to $T \gtrsim 10^9$ K leading to

$$^{12}C + {}^{12}C \to {}^{24}Mg + \gamma \to {}^{23}Na + H \to {}^{23}Mg + n$$
$$\to {}^{20}Ne + {}^4He \to {}^{16}O + {}^4He + {}^4He$$

and at even higher T's:

$$^{16}O + {}^{16}O \to {}^{32}S + \gamma \to {}^{31}P + H \to {}^{31}S + n \to {}^{28}Si + {}^4He$$
$$\to {}^{24}Mg + {}^4He + {}^4He$$

The process of forming successively heavier elements through exothermic reactions stops @ $A \simeq 56$ (^{56}Fe, ^{56}Ni), where the binding energy (per nucleon) reaches a maximum.

The binding energy of a nucleus is the result of the balance between attractive nuclear forces and the repulsive Coulomb force. Although nuclear forces are stronger, they have a short range. As a result, nuclear binding energies increase only linearly with the number of nucleons (protons/neutrons) whereas the Coulomb repulsion force increases quadratically with the number of protons. Since nuclear forces are on average more attractive between ~~neutrons~~ neutrons and protons than between identical nucleons, ~~the~~ N and Z tend to be roughly equal in stable nuclei, especially for light nuclei ($A < 56$) where Coulomb repulsion is not significant. Then adding nucleons one increases the binding energy. However as A increases beyond 56, electrostatic repulsion becomes important and more neutrons are needed in order to have more nuclear attractions to compensate for the Coulomb repulsion of protons. Therefore the valley of stability shifts to $N > Z$ (for $A \gtrsim 56$) and binding energy per nucleon decreases. Eventually, when $Z \geq 82$ there are no stable nuclei, ~~there~~ due to its short range the nuclear force cannot fight against ~~the~~ Coulomb repulsion anymore.

Therefore beyond $A = 56$ ~~it is~~ it is not energetically favourable to synthesize heavier elements. Such nuclei are produced by neutron capture in supernovae explosions (where ambient neutron flux is large).

One final comment. When ^{12}C is present a more efficient ~~H~~ hydrogen burning process may take place (at high enough T). This is the CNO cycle, where carbon is used as a cathalyst to convert 4 protons into ^4He. CNO cycle is more efficient than pp above $T \sim 2 \times 10^7 K$