Recombination, Decoupling & Residual Ionization

After BBN the universe is mainly made of protons, ^4He nuclei, $\bar{e}$, ~~ν's~~, and decoupled γ's, with all massive particles being non-relativistic at $T < 0.5$ MeV ($= m_e$). These interact with one another through different processes,

- Charged particles interact through Coulomb scattering
- γ's interact with charged ~~with~~ through Compton scattering (which when photon momentum becomes less than particle mass is Thomson scattering), Bremsstrahlung and recombination ($p+e \leftrightarrow H+\gamma$)

We shall see that three events happen at about the same time for cosmological parameters of interest.

Let's start with an equilibrium description of recombination, assuming in addition that recombination happens by protons and electrons forming hydrogen at the ground state (creating a photon with 13.6 eV) – we'll see later this is not quite right.

Since everything is non-relativistic we have

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right] \qquad i = e, p, H$$

and since REC: $e+p \leftrightarrow H+\gamma \Rightarrow \mu_e + \mu_p = \mu_H$

like for BBN we can use this relationship between μ's to write n_H

$$n_H \underset{\substack{m_p \simeq m_H \\ \text{in prefactors}}}{\uparrow=} \frac{g_H}{g_e g_p} n_e n_p \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp[B/T]$$

where $B = 13.6$ eV $\simeq m_p + m_e - m_H$

Let's work in terms of fractional ionization X, since $n_p = n_e$ by charge neutrality we have $X_p = X_e$ and using $n_H + n_p = n_B$ (baryon # density)

$$X_H = \frac{n_H}{n_B} = \frac{n_B - n_p}{n_B} = 1 - X_p = 1 - X_e$$

$$\Rightarrow \begin{cases} n_H = (1-X_e)\, n_B = (1-X_e)\, n_\gamma\, \eta \\ n_p = n_e = X_e\, n_B = X_e\, n_\gamma\, \eta \end{cases}$$

$\eta = 2.68 \times 10^{-8}\, \Omega_B h^2$ is baryon to photon ratio

$$\Rightarrow \boxed{\frac{1-X_e}{X_e^2} = \underbrace{\frac{g_H}{g_e\, g_p}}_{=1} \underbrace{\left[\frac{2\zeta(3)}{\pi^2} T^3\right]}_{n_\gamma} \left(\frac{m_e T}{2\pi}\right)^{-3/2} \eta\, e^{B/T} = \frac{2^{5/2}}{\pi^{1/2}} \zeta(3) \left(\frac{T}{m_e}\right)^{3/2} \eta\, e^{B/T}}$$

(This is the "Saha equation")

since $g_e = g_p = 2$

$g_H = 4$ (for ground state including hyperfine structure)

Let's define recombination when 90% of the atoms have been formed, then T_{REC} corresponds to $X_H = 0.9$ and $X_e = 0.1$,

$$\Rightarrow \frac{0.9}{(0.1)^2} = 90 = 2.7 \times 10^{-16}\, \Omega_B h^2\, T_1^{3/2}\, e^{13.6/T_1} \qquad T_1 \equiv \frac{T_{REC}}{1\,\mathrm{eV}}$$

$$\Rightarrow (\Omega_B h^2)^{-1}\, T_1^{-3/2}\, e^{-13.6/T_1} \simeq 3 \times 10^{-18}$$

taking ln's and iterating we get : $\frac{1}{T_1} \simeq 3 - 0.07 \ln(\Omega_B h^2)$

$$\Rightarrow (1+z)_{REC} = \frac{T_{REC}}{T_0} = 1367\, [1 - 0.024 \ln \Omega_B h^2]^{-1}$$

Therefore ($\Omega_B h^2 \simeq 0.02$) $\quad (1+z)_{REC} \simeq 1300$, $T_{REC} \simeq 0.3\,\mathrm{eV}$.

As in BBN, due to the large number of photons around (tiny η), recombination into atoms is delayed from naive estimate 13.6 eV down to 0.3 eV.

Again, recall that this calculation assumes that reaction rates are fast enough so that equilibrium is justified - let's check this -

The recombination cross section is roughly,

$$\sigma \sim \lambda^2_{\text{Compton}} \sim \frac{1}{p^2} \sim \frac{1}{v^2} \quad \Rightarrow \quad \sigma v \sim \frac{1}{v} \sim \frac{1}{T^{1/2}}$$

Putting numbers $\langle \sigma v \rangle \sim 4.7 \times 10^{-24} \left(\frac{T}{1\text{eV}}\right)^{-1/2} \text{cm}^2$

(Thomson cross section is $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 = 0.665 \times 10^{-24}\,\text{cm}^2$ for electrons)

Then $\Gamma = n_e \langle \sigma v \rangle = X_e\, \eta\, n_\gamma \langle \sigma v \rangle$

In order to evaluate the interaction rate Γ we need the ionization fraction $X_e(T)$. To do this let's assume we are in EQ and use worst case scenario where $X_e \ll 1$ [remember, we want to check whether we are in EQ, but if the answer ends up being that we are, the calculation is self-consistent]. Then, equation for X_e above gives for $X_e \ll 1$:

$$X_e(T) \approx 10^4\, \eta^{-1/2}\, T_1^{-3/4}\, e^{-6.8/T_1} \qquad T_1 = \frac{T}{1\text{eV}}$$

$$\Rightarrow \quad \Gamma \sim 7.2\, \frac{1}{\text{sec}}\, T_1^{7/4}\, e^{-6.8/T_1}\, (\Omega_B h^2)^{1/2}$$

Taking the universe as matter dominated, $H^2 \sim T^3$, then the expansion rate is

$$H \simeq 9 \times 10^{-13}\, \frac{1}{\text{sec}}\, (\Omega_M h^2)^{1/2}\, T_1^{3/2}$$

Then EQ condition ($\Gamma = H$) reads

$$T_1^{1/4}\, e^{6.8/T_1} = 8 \times 10^{12} \left(\frac{\Omega_B}{\Omega_M}\right)^{1/2}$$

Using the old trick, taking ln's:

$$\frac{1}{T_1} \simeq 4.3 - 0.07 \ln \frac{\Omega_M}{\Omega_B} \quad \Rightarrow \quad (1+z) \simeq 980 \left(1 - 0.017 \ln \frac{\Omega_M}{\Omega_B}\right)^{-1}$$

Since $\Omega_B h^2 \sim 0.02$ from BBN, using $\Omega_M/\Omega_B \sim 10$ we have

Freeze-out of recombination is @ $T_F \simeq 0.24\,\text{eV}$

Note this is lower than $T_{REC} \approx 0.3\,\text{eV}$, so we were in principle OK assuming equilibrium.

- Since recombination is frozen @ this T, we can calculate what is the freeze-out ionization fraction left, $X_e(T_F)$

$$X_e(T_F) \simeq 10^4\,\eta^{-1/2} \left(\frac{T_F}{1\,eV}\right)^{-3/4} e^{-6.8/(T_F/1eV)} \underset{\substack{\uparrow \\ \text{using} \\ \Gamma=H \text{ condition} \\ \text{above}}}{=} 10^4\,\eta^{-1/2} \left(\frac{T_F}{1eV}\right)^{-1} (8\times10^{12})^{-1} \left(\frac{\Omega_M}{\Omega_B}\right)^{1/2}$$

$$\Rightarrow X_e(T_F) \simeq 7.4\times10^{-6} \left(\frac{T_F}{1eV}\right)^{-1} \frac{\sqrt{\Omega_m h^2}}{\Omega_B h^2}$$

For $T_F \simeq 0.24$ eV $\Rightarrow$ $\boxed{X_e \sim 3\times10^{-5}\,\frac{\sqrt{\Omega_m h^2}}{\Omega_B h^2}}$

Therefore, a fraction of order 10^{-4} of $\bar{e}$ and p remain free.

[These e and p remain in EQ with each other through Coulomb scattering.
Recall that $\sigma \sim \frac{\lambda^4 E^2}{(p^2+m^2)^2}$, $m=0$ for γ (Electromagnetic interaction), $\lambda \sim e$, $E \sim m$
and $p \sim mv$ $\Rightarrow$ $\sigma \sim \frac{e^4}{m^2 v^4} \sim \frac{e^4}{T^2}$

$$\Rightarrow \frac{1}{\Gamma} = \frac{1}{n_e \sigma v} \sim \frac{1}{n_e}\left(\frac{e^2}{T}\right)^{-2}\left(\frac{m}{T}\right)^{1/2} \simeq 1.3\text{ sec}\left(\frac{T}{1eV}\right)^{-3/2}(X_e\,\Omega_B h^2)^{-1}$$

smaller than expansion rate $H^{-1} \simeq 1.1\times10^{12}$ sec $\left(\frac{T}{1eV}\right)^{-3/2}(\Omega_m h^2)^{-1/2}$
even though $X_e \sim 10^{-5}$. Therefore $t_{ee} \ll H^{-1}$]

Now we need to go back and check the approximations that have been made. Although we checked that equilibrium holds down to T_{REC}, the other main assumption is that REC happens to the ground state. We have the reaction

$$p+e \rightarrow H+\gamma \qquad E_\gamma = 13.6\text{ eV}$$

and the inverse: $\gamma + H \rightarrow p+e$

The question is: as T decreases the # of γ's in the bath with $E \geq 13.6$

decreases, but the number of given γ's we get out of recombinations increases. It turns out that @ $T \sim 0.8\,eV$ and below there would be more γ's around from recombinations (which are not included in the calculation), these γ's can ionize hydrogen atoms formed a bit earlier. As a result of this, recombination to the ground state is not very effective in practice.

The way recombination then proceeds is by excited states, not to the ground state, e.g

$$e + p \rightarrow \underbrace{H^*}_{\substack{2P,\,or \\ 2S\ state}} + \gamma_1 \longrightarrow H + \gamma_1 + \gamma_2$$

The two mechanisms are:

<u>to 2P then to GS</u>: (Lyman α = 1216 Å). Each γ has less energy than 13.6 eV and they get redshifted away from 1216 Å resonance by expansion of the universe

<u>to 2S then to GS</u>: this is forbidden by E and L conservation for 1 γ. So it can only be done with 2 γ's.

The key here is that inverse processes are much slower and thus REC proceeds in a non-equilibrium fashion. As a result of this REC happens at a slower rate than predicted by EQ calculation, but interestingly, the numbers we obtained above are pretty close to the correct ones! [show X_e plot].

There is a third process (apart from REC and freeze out of ionization fraction) that happens around the same time: <u>decoupling</u> (DEC). After X_e drops to 10^{-4}–10^{-5} the Compton and Bremsstrahlung processes become negligible, the only interaction between γ's and free

electrons is through Thomson scattering, which is like Compton scattering but without energy transfer, the γ's simply perform a random walk. [The reason for this is that as we discussed above $\sigma_{Compton} \sim \sigma_T \frac{T}{m_e} \ll \sigma_T$]
(Compton works up to $T \gtrsim \text{keV}$)

As the number density of charged particle decreases, even this interaction rate freezes out, at some $T = T_{dec}$. After that, γ's propagate in straight lines (except for when the universe ionizes again at $z \sim 20$-10 due to formation of stars): this is the CMB radiation we see today.

Let's calculate T_{DEC}:

$$\Gamma = \sigma_T n_e = \sigma_T X_e n_B = \sigma_T X_e \eta n_\gamma = (\Omega_B h^2)^{1/2} \left(\frac{T}{1\text{eV}}\right)^{9/4} e^{-\frac{6.8}{(T/1\text{eV})}} \frac{1}{\text{sec}}$$

$$H = 2 \times 10^{-13} (\Omega_M h^2)^{1/2} \left(\frac{T}{1\text{eV}}\right)^{3/2}$$

$$\overset{H=\Gamma}{\Rightarrow} \quad T_1^{-3/4} e^{6.8/T_1} = 10^{12} \left(\frac{\Omega_B}{\Omega_M}\right)^{1/2}$$

Solving as usual, $\quad \frac{1}{T_1} = 3.2 + 0.07 \ln \frac{\Omega_B}{\Omega_M}$

$$\Rightarrow \quad T_{DEC} \simeq 0.26 \text{ eV} \qquad (1+z)_{dec} \simeq 1100$$

(WMAP 1st year gives $z_{dec} = 1088^{+1}_{-2}$)

After this, for $T < T_{DEC}$, $T_\gamma \sim 1/a$ whereas $T_{\substack{matter\\(atoms)}} \sim \frac{1}{a^2}$, the adiabatic cooling for non-relativistic case.

For the ionized matter, however, it is a bit different story. Since the number of photons is so large compared to e^-, the scatterers of e^- (the photons) give rise to a short mean free path:

$$\underset{\substack{\text{mean free}\\\text{path of } e^-\\\text{(due to } \gamma\text{'s)}}}{\lambda_e} \sim \frac{1}{n_\gamma \sigma_T} \ll \frac{1}{n_e \sigma_T} = \lambda_\gamma$$

and this is strong enough to keep e^- in EQ with photons up to $T \sim 60°K$ ($z \simeq 20$)! Photons do not care, only <u>very very few</u> of them participate in this.