Homework Set #2 (Due 10/26 in Class)

1.

a) Calculate the increase (in the sudden approximation) in temperature during $\mu^+\mu^-$ annihilation. What is temperature of the photons at this energy scale (after annihilation)? What about the neutrino temperature?

b) Calculate the energy densities of $\nu\bar{\nu}$ (assume 3 families), photons, and e^+e^- at neutrino decoupling and when $T_\gamma \sim 0.1$MeV.

c) Show that at high enough energies the universe cannot be in thermal equilibrium, assuming the conventional interactions.

2.

a) Show that for a cold relic with cross section σ_0,

$$\Omega h^2 \sim \Big(\frac{\sigma_0}{10^{-37}\text{cm}^2}\Big)^{-1}, \tag{1}$$

so that the annihilation cross section should be comparable to weak interactions in order to provide a siginificant contribution to the energy density today.

b) Assume a hypothetical massive (stable) particle decouples when relativistic at $T \sim 300$ GeV. What is the bound on its mass from requiring that its density today obeys $\Omega h^2 \lesssim 1$?

3. Massive galaxies are found to host supermassive ($M \sim 10^7 M_\odot$) black holes at their centers. Assuming all galaxies in the universe host such black holes, calculate the entropy in black holes and compare it to the entropy of the CMB. The entropy of a black hole is $S = A/4$, where A is the horizon area in Planck units.

4. What happens to ^{4}He production during nucleosynthesis if

a) the baryon density were larger,

b) the coupling constant of weak interactions were larger,

c) Newton's gravitational constant were larger during nucleosynthesis than today

d) the CMB temperature were much smaller than 3^oK,

e) the universe contained many more neutrinos than antineutrinos. What about deuterium in this case?

f) Suposse the baryon to photon ratio in the universe were $\eta \sim 1$. Nucleosynthesis would then be qualitatively different. Explain why.

5.

a) Calculate the mean free path of photons before and after recombination, assuming that there is a residual reionization of $X = n_e/n = 1.21 \times 10^{-5}\sqrt{\Omega_m}/(h\Omega_B)$. Assume instantaneous recombination.

b) Compare the result in a) to the Hubble radius.

c) Calculate the optical depth $K(t_0, t) = \int_t^{t_0} n_e(t)\sigma_T c dt$ as a function of redshift, before and after recombination. What is the physical interpretation of K?

d) Explain why decoupling of photons, recombination of atoms, and the setting of residual ionization happen all at about the same time in our universe. Why do these happen during matter domination?