1) Check that the following equation

\[ y' = -\frac{y \cos(x) + 2xy^2}{\sin(x) + 2x^2y - 1} \tag{1} \]

is exact, and find its solution for boundary condition \( y(\pi/2) = 1 \).

2) Find the general solution of

\[ y'' - 3y' + 2y = 5e^{2x} \tag{2} \]

3) Use Frobenius to find two independent solutions of

\[ y'' - xy = 0 \tag{3} \]

and write explicitly the first three terms in the series of each solution.

4) Find the steady-state temperature distribution of a rectangular plate with sides \( L_x = 20 \) and \( L_y = 40 \), with the following boundary conditions,

\[
\begin{align*}
T(x = 0, y) &= 0 \\
T(x = 20, y) &= \sin(\pi y) \\
T(x, y = 0) &= 0 \\
T(x, y = 40) &= 0
\end{align*}
\]

5) A cylinder of radius \( a \) and height \( 2h \) centered about \( z = 0 \) is held to temperature \( T = 0 \) at its wall, \( T = T_0 J_0(\alpha_0 \xi_a) \) at its top face \( (z = h) \) and \( T = -T_0 J_0(\alpha_0 \xi_a) \) at its bottom face \( (z = -h) \). Find the temperature distribution inside.

6) One sphere of radius \( a \) is held at temperature \( T = T_0 \), while a second sphere of radius \( 2a \) concentric with the first is held to \( T = T_0 \cos^2(\theta) \). Find the temperature distribution everywhere in space.
7) Let the complex function \( f(z) \) be analytic, \( f = u + iv \) with \( u, v \) real functions of \( x, y \), and \( z = x + iy \).

i) Show that if the real part of \( f \) is constant, then \( f \) must be constant.

ii) Show that if the absolute value of \( f \) is a constant, then \( f \) must be constant.

8) Let the complex function \( f(z) \) be analytic, with its real part given by \( u(x, y) = 3x^2 - 3y^2 \), where \( z = x + iy \).

i) Find its imaginary part \( v(x, y) \) and rewrite the function \( f(z) \) in terms of \( z \).

ii) Calculate

\[
\oint_C \frac{f(z)}{(z - i)}
\]

where \( C \) is a counterclockwise path given by \( |z| = 2 \).

9) Consider the function

\[
f(z) = \frac{z}{1 - \cos z}
\]

i) Find all the poles of \( f(z) \) and identify their order.

ii) Calculate

\[
\oint_{C} f(z) \, dz
\]

where \( C \) is a counterclockwise path given by \( |z| = 1 \).

ii) Calculate

\[
\oint_{C} f(z) \, dz
\]

where \( C \) is a counterclockwise path given by \( |z - 2\pi| = 1 \).